3 Functions, equations and geometry II

Depending on your background, some of these exercises may be easy, while others may be more difficult. We simply ask you to do your best with these problems, within the limits of the time allocated for the course.

Read **these reminders** before you begin!

(keeping track)

(CONSUMER TEST RANKING)

Suggest a systematic approach for ranking different products, e.g. computers, dishwashers etc., to determine the best buy. You may need to create your own concrete examples to develop your ideas, and to explain how you are thinking.

(investigating the abstract)

(THE ARITHMETIC AND GEOMETRIC MEAN)

a) Investigate the relation between the arithmetic mean and geometric mean. Begin to try out in an experimental way.

arithmetic mean $\frac{x_1 + x_2 + \cdots + x_n}{n}$

geometric mean $\sqrt[n]{x_1 x_2 \cdots x_n}$

b) If you found some interesting property, try to prove it. Otherwise you will not know if it is always true or not!

HINT try simple cases first!

(investigating the world)

(EMPIRICAL CURVE FITTING)

For two related physical variables the following relationship has been measured:

time	distance
88.0	57.9
224.7	108.2
365.3	149.6
687.0	228.07

time	distance
4332	778.434
10760	1428.74
30684	2839.08
60188	4490.8
90467	5879.13

Investigate and suggest a mathematical equation describing the relation between these two quantities. **We ask you to NOT use a graph drawing calculator or other software, with automatic curve fitting. We want you to experience the problem of manually searching for a function yourselves. Preferably use the Mathematica functions suggested below for plotting.** Otherwise, feel free to use your creativity.

Explain how you did to find the model, and motivate your choice. Is the fit of your equation good? How can deviations from the known table entries be justified?

HINT: Make sure you get started by doing something. Plot the points, plot some functions and explore all possibilities you can think of. What you definitely shouldn't do is to try to look up an answer, it is your own exploration that is important. Try to adopt an investigative attitude!

You don't have to use Mathematica for this problem, but if you want to plot the data points in Mathematica you can use `ListPlot`. If you want to superimpose a function and the points in the same diagram you can use `Show` with `Plot[your function]` and `ListPlot[the data]` as arguments. Look in the Mathematica help system for extensive documentation and examples.

```
p1 = ListPlot[{{88.0, 57.9},
{224.7, 108.2}, {365.3, 149.6},
{687.0,228.07}, {4332, 778.434},
{10760, 1428.74},
{30684,2839.08}, {60188,
4490.8}, {90467, 5879.13}}];
p2 = Plot[0.04 x, {x, 0,
100000}, PlotStyle -> Red];
Show[p1, p2]
```

(We don't say now where the data comes from because it is a good exercise to practice purely empirical modelling. We will tell you afterwards.)

(BOKEH)

(Voluntary - but try to understand the problem so you can appreciate the follow-up)

As you may know, some photos, especially portraits, sometimes have a nice blurred background. Since some photos/cameras seem to give this and others not, we would

like you to (theoretically) investigate what it is that creates the effect, and what factors influence the strength of this effect and in what way.

a) Investigate what conclusions you can reach based on your everyday knowledge and anything that you may be able to figure out from that. To get you started we can remind you that if you have a magnifying glass, parallel rays will at a certain distance behind the lens concentrate into one point.

b) Can you draw further conclusions if you make use of **the thin lens formula**?

(designing)

(CURVE FITTING AND COMPUTER GRAPHICS)

This is a reading exercise.

- **A short note on curve fitting**
- **Bezier curves in computer graphics**. The Bezier curves were first invented by Pierre Bézier at Renault (automotive modelling). This text is quite characteristic for the use of mathematics in computer science and engineering, and also gives a (limited) impression of the area of computer graphics and curve modelling.

When the text talks about a point, it refers to a two or three dimensional vector with the x,y and z coordinates. Such vectors can easily be added and multiplied by a scalar (a single number). Ask if you need further explanation! See also the short note on vector arithmetic at the end of this module.

Make an attempt to read and understand the text, especially the first part. Don't worry if you have difficulties in understanding some parts - simply try to read and see how far you get and what difficulties you run into. Technical texts are never easy, and whatever it is that you manage to understand is good!

As the answer in a concise way explain the parts you understand. Remember how we discussed about the unit conversions. In whatever you explain, do not only explain something around the issue, but try to explain the core of the matter as clearly as possible. Doing this forces you to think about what the main concepts really are.

(thinking)

(REASONING - DIFFERENT KINDS OF REASONING)

Try to characterize what kind of reasoning is used in the following conclusions:

- I have parked my bike here many times, so I think it's safe.
- All men are mortal. Socrates is a man. Therefore, Socrates in mortal.
- The lawn is wet. I suppose it must have rained.
- From the figure we can see that the area of the circle is clearly less than 4r^2, and also more than 2r^2. So let's say 3*r^2.
- We can see in this graph that those with more education are much less likely to become unemployed in the future. So therefore, we should encourage everyone to get a proper education.
- Our state has the highest crime rate in the country. And our city has the highest crime rate in the state. So we have the highest crime rate in the country.
- If there are no more than five possibilities and I check all of them, then I will know the answer.
- My very healthy grandfather smokes two packages a day and he is 95 years old => smoking is harmless.
- It is raining. I have an umbrella => I will take the umbrella.

The most important distinction is between *deductive* reasoning (where the conclusion is certain given the premises), and *plausible* reasoning (where the conclusion may be probable but is not certain), so primarily try to categorize the reasoning in these two groups. Common forms of plausible reasoning are *inductive* and *abductive* reasoning (not exhaustive).

(REASONING - NATURE OF PROOFS)

A proof can be seen as a deduction from known (or assumed) premises (input) to a conclusion (output). So it's like a function in this respect - from what we already know to something new that we didn't know. The most basic assumptions we begin with are called *axioms* (obvious truths), but often other generally known results are accepted as "input" to other proofs. What premises the proof is using is an important aspect of the proof, and that can be quite different for different proofs of the same thing! It can sometimes be difficult to sort out exactly what basic assumptions a proof depends on since we often tend to use what we already know quite freely.

a) For a nice example, see this **discussion on proofs for the sum of the angles in a triangle**. Answer this question by explaining in your own words what the premises are in the two proofs presented.

b) (voluntary) Have a look at **some proofs of Pythagoras' theorem**, and try to understand what the premises are also here.

(MODELLING/REASONING - ORIGIN AND MOTIVATION OF FORMULAS)

This problem is to make you aware of where formulas come from and to what extent you should believe in them.

For each equation below discuss:

a) how do we know it is true (or at least useful), and

b) is it exact or is it some kind of approximation)?

The equations:

- a^2+b^2=c^2 (Pythagoras' theorem)
- population= C * a^t (C and a are constants, t is the time in years)
- F= G m1 m2 / r^2 (gravity between two bodies)
- stock index= 2045+0.0034 t (trend analysis)
- 100*weight+length < 320 (max allowed parcel size for a postal service)
- #presentStudents + #absentStudents = #allStudents (for a class)
- air drag force = C* v^2 * A (air drag for a moving object, C is a constant depending on the shape but not the size, v velocity, A cross section area)

If you can, try to categorize the expressions in different groups. Spend a moderate time on this exercise, a short comment on each equation and maybe a few general observations is sufficient.

(mathematical knowledge)

(A NOTE ABOUT VECTOR ARITHMETIC)

Some notes (If you need help please contact a teacher)

If you like you can also read at **https://en.wikipedia.org/wiki/Dot_product**

(finally...)

(SELF-CHECK)

- Have you answered all questions to the best of your ability?
- Is all the required information on the front page, is the file name correct etc.?
- Anything else you can easily check? (details, terminology, arguments, clearly stated answers etc.?)

Do not submit an incomplete module! We are available to help you, and you can receive a short extension if you contact us.