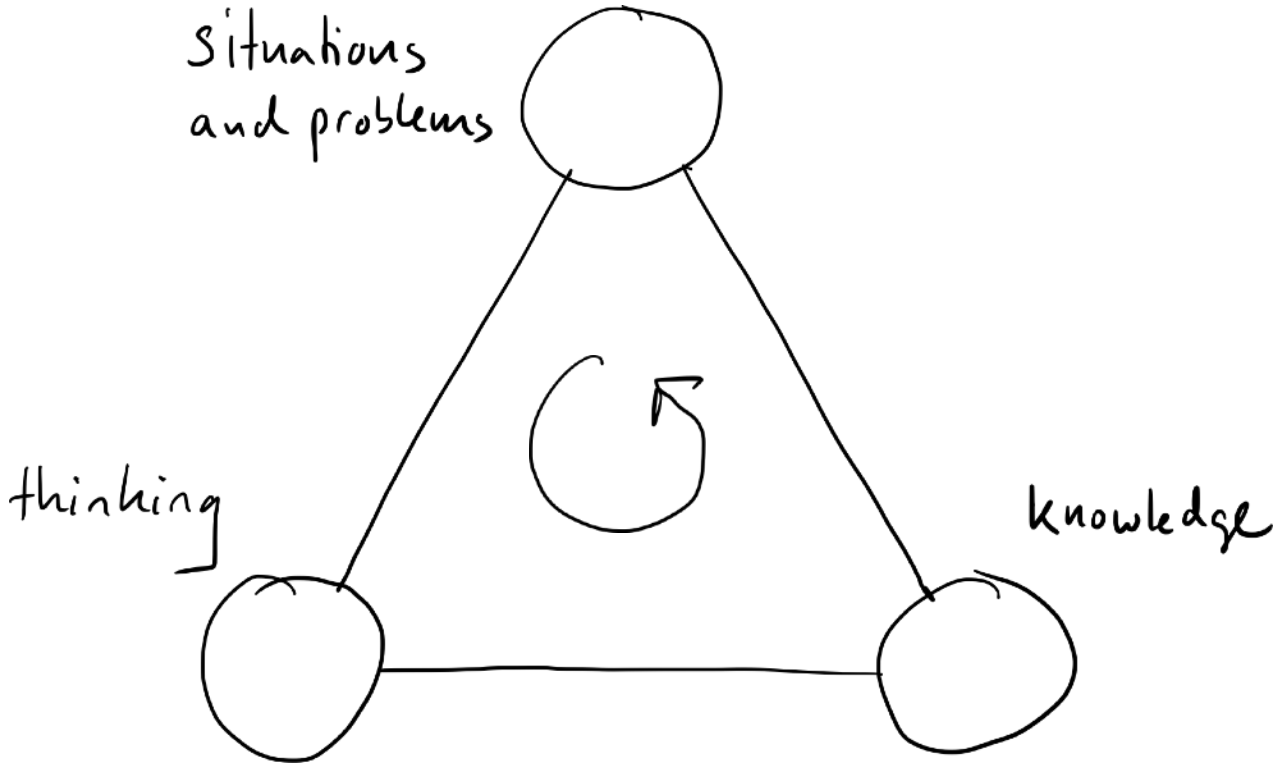
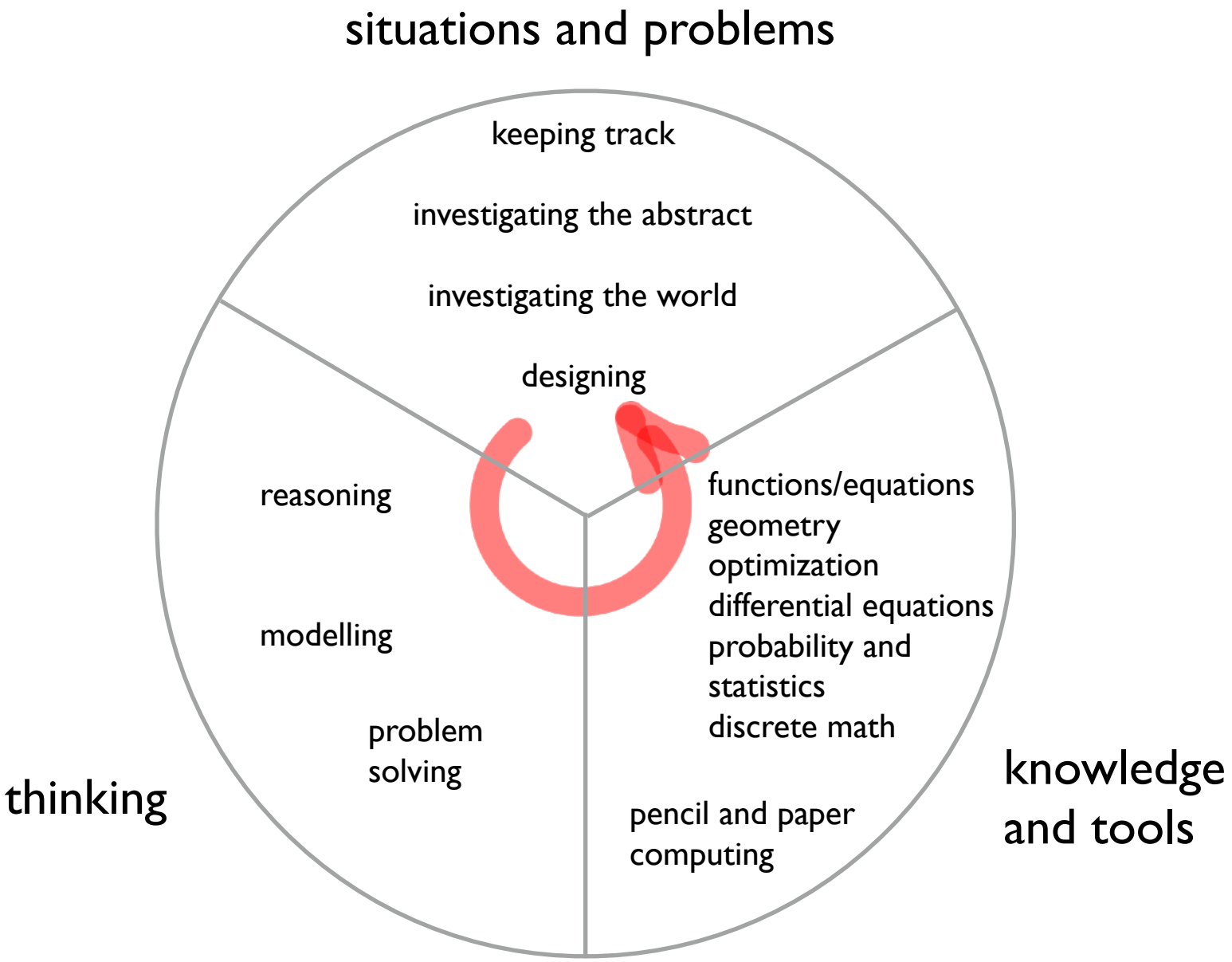
Applied mathematical thinking
Course summary

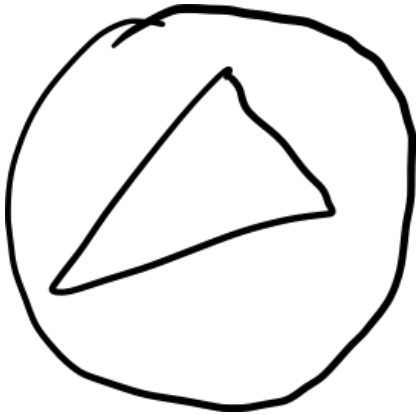
Dag Wedelin


situations
and problems
thinking
knowledge


situations and problems
keeping track
investigating the abstract
investigating the world
designing
reasoning
modelling
problem solving
thinking
functions/equations
geometry
optimization
differential equations
probability and statistics
discrete math
pencil and paper computing
knowledge and tools

mathematical reasoning

We are naturally able to imagine and reason about abstract mathematical concepts!



In our mathematical thinking we can therefore connect common sense and the use of mathematics.

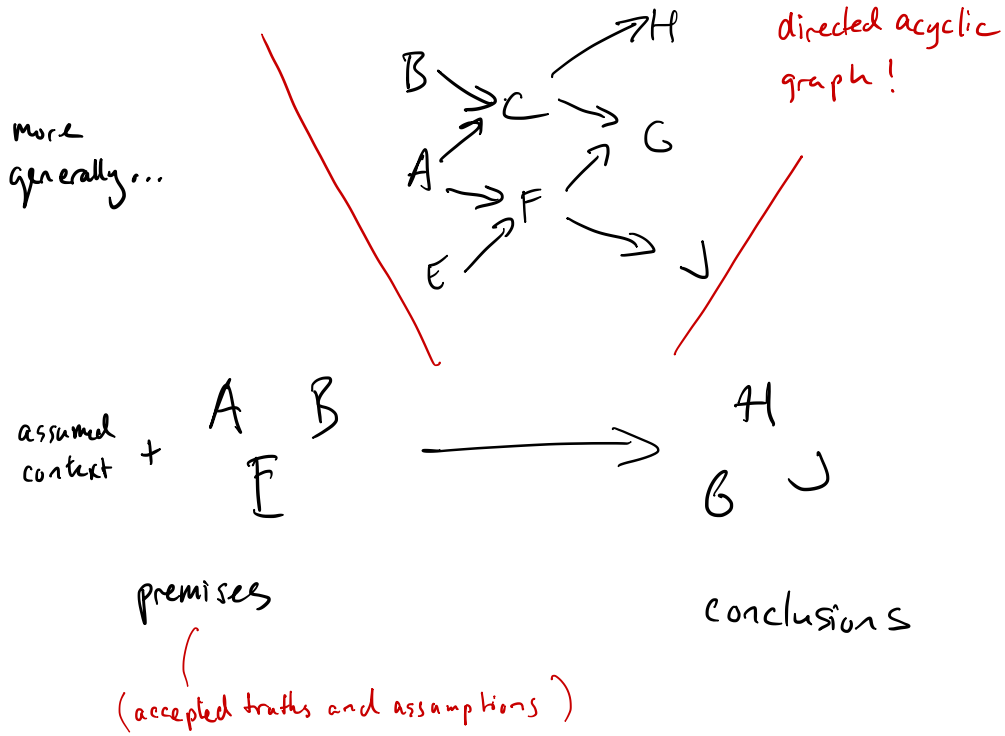
On the nature of reasoning

the importance of precision!
(*exactly what is…*)

deductive and plausible reasoning

premises and conclusions

necessary and sufficient conditions



Discovery comes first

1 Discovery is usually easier, and therefore mostly comes first.

Make observations and create conjectures

you are able to discover a lot!

---

2 Some things may then be quite easy to confirm or prove

Some things can be very difficult to prove

you don't know in advance which is which!

the discovery phase is usually not documented and is therefore not visible!

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you don't know in advance which is which!

think about this also when you study mathematics and other subjects!

the discovery phase is usually not documented and is therefore not visible!

Towards better explanations…

Statements are wrong -
be careful to never say anything that is wrong!

Correct, but superficial -
connections between statements can be improved

Efficient use of all kinds of reasoning in well connected arguments and explanations

"Mathematics is the art of explanation"

(P. Lockhart)

what have you learned about mathematical reasoning?

simple experimentation
=> nice discoveries !

formal notation not
necessary

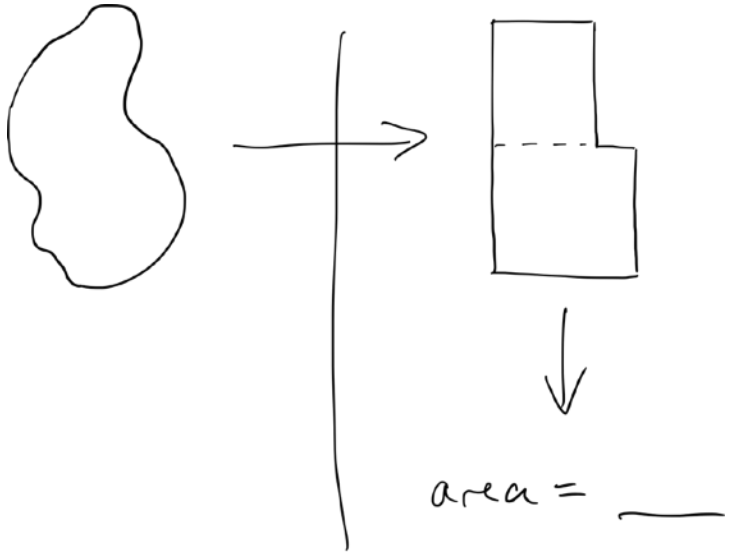
what have you learned about mathematical reasoning?

the course highlighted the
whole process of exploration,
that is usually invisible

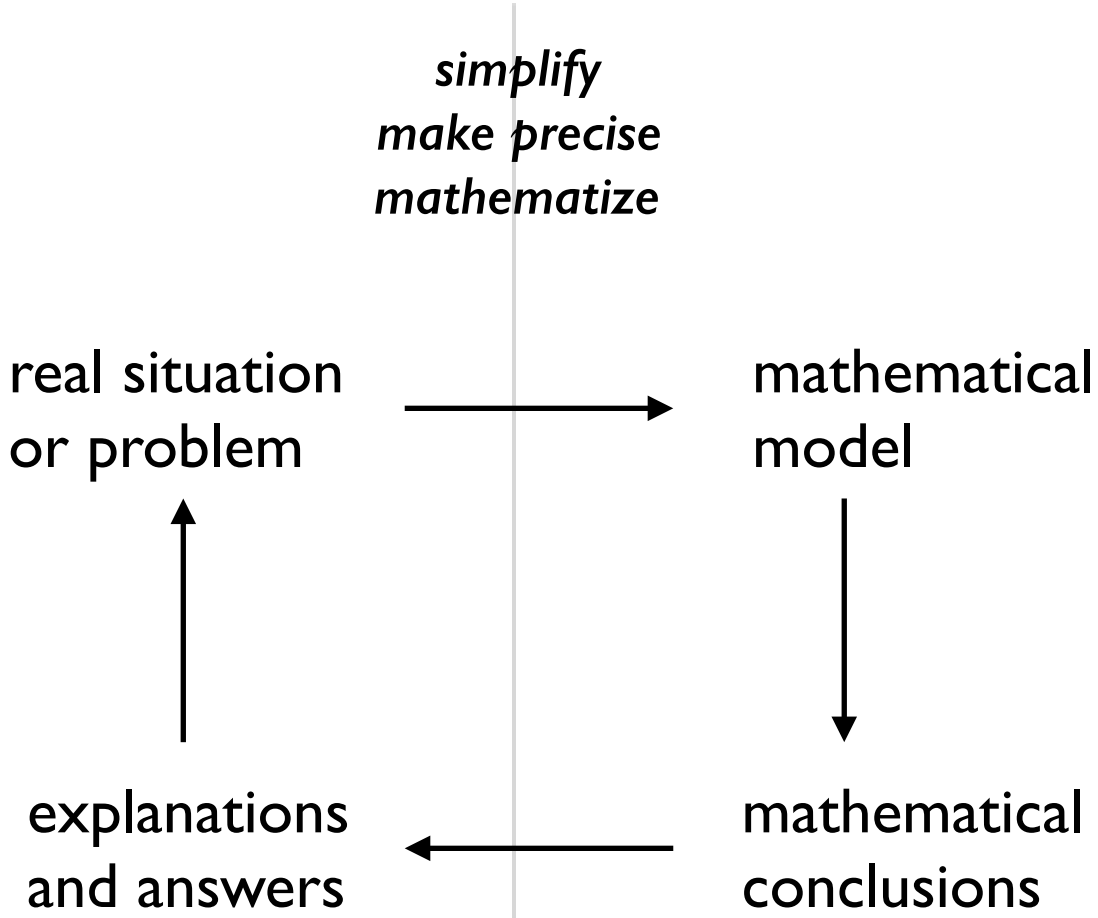
(2017)

models and modelling

Why models?



"a convenient way to represent reality so that we more easily can draw conclusions about it"



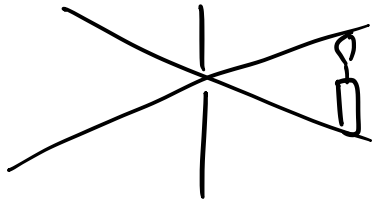
a special case of designing…

A critical decision: selecting the modelling approach!

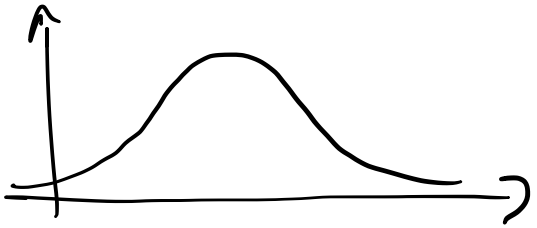
$y = x^2$

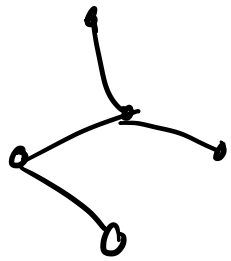
$\min cx$
subject to $Ax = b$

$2x^2 - 4x + 1 = 0$



$p' = ap$





what have you learned about mathematical modelling?

mathemtical modelling can be universally applied

modelling to standard problems

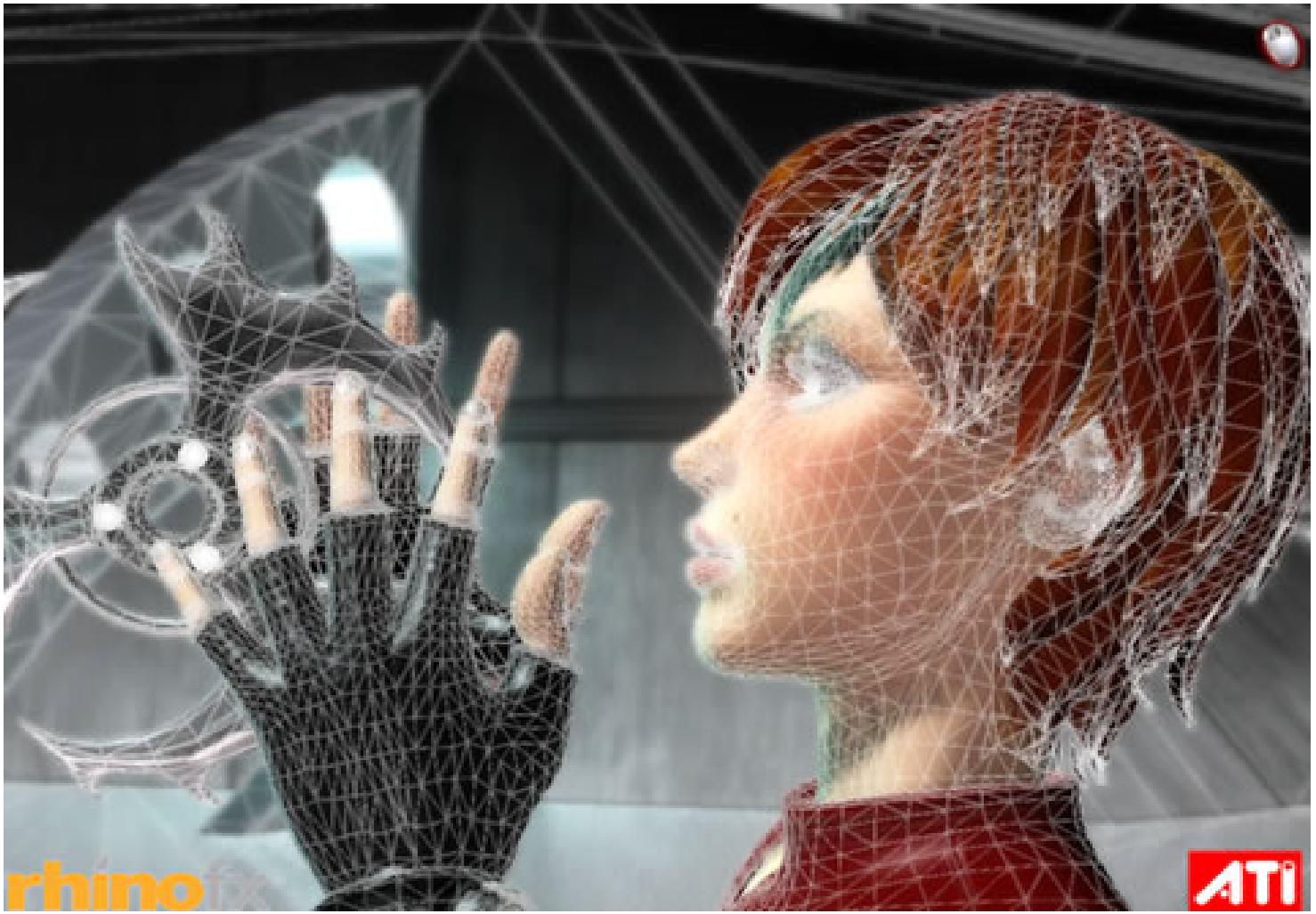
what have you learned about mathematical modelling?

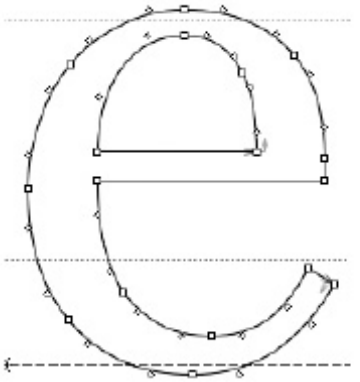
try other options

start anywhere with something simple!

(2017)

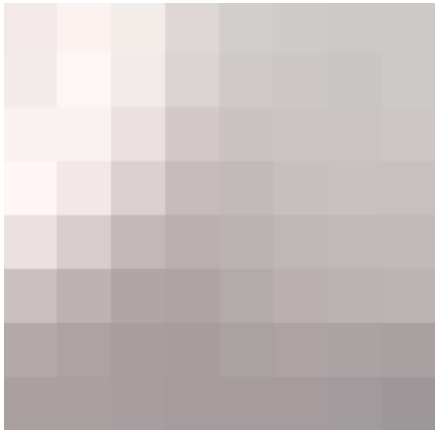
The math is not always visible!





A glyph drawing in TrueType format, using quadratic curves. Note that it has more control points than the PostScript counterpart.

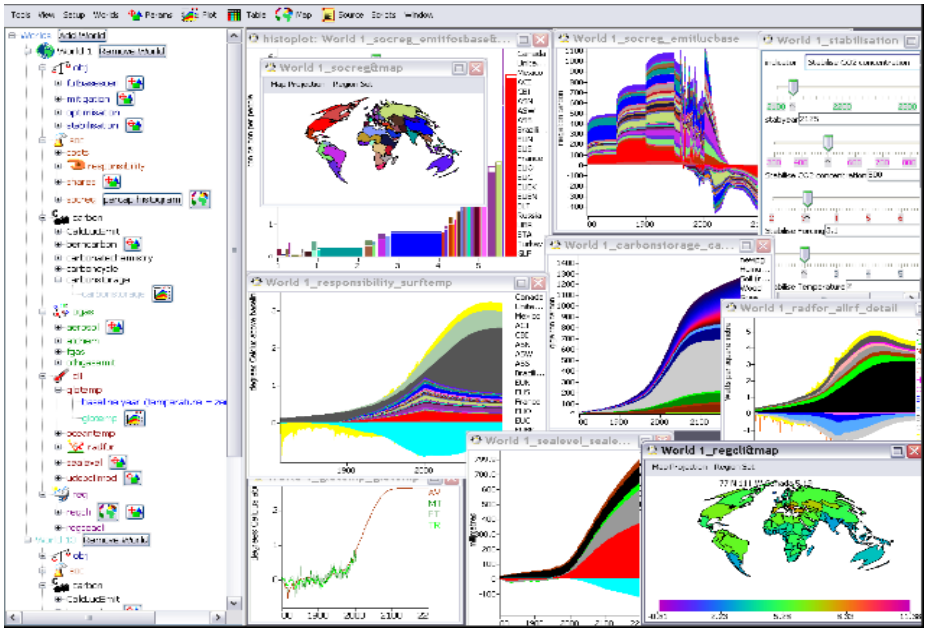
jpeg compression

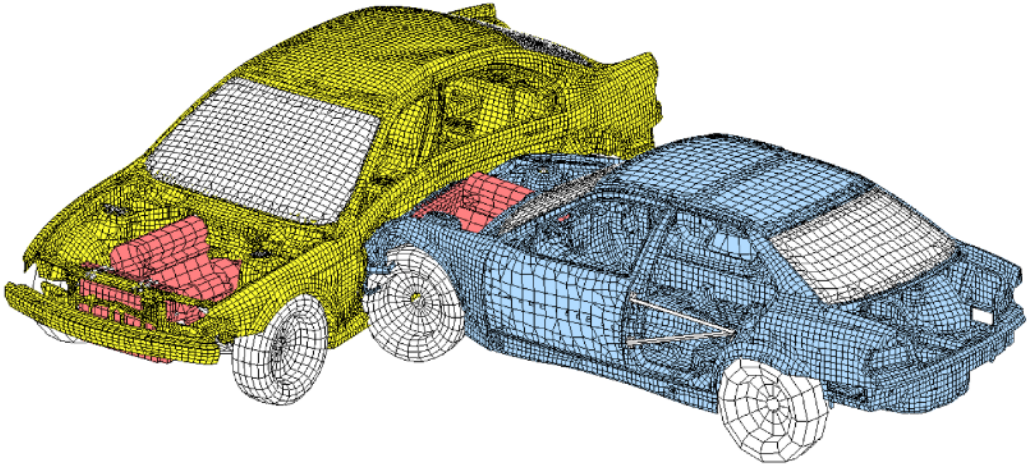


$$
\begin{aligned}
Y' &= 16+ (65.481 \cdot R'+ 128.553 \cdot G'+ 24.966 \cdot B') \\
C_B &= 128+ (-37.797 \cdot R'- 74.203 \cdot G'+ 112.0 \cdot B') \\
C_R &= 128+ (112.0 \cdot R'- 93.786 \cdot G'- 18.214 \cdot B')
\end{aligned}
$$

$$
G_{u,v} = \sum_{x=0}^{7} \sum_{y=0}^{7} \alpha(u)\alpha(v) g_{x,y} \cos\left[\frac{\pi}{8}\left(x+\frac{1}{2}\right)u\right] \cos\left[\frac{\pi}{8}\left(y+\frac{1}{2}\right)v\right]
$$

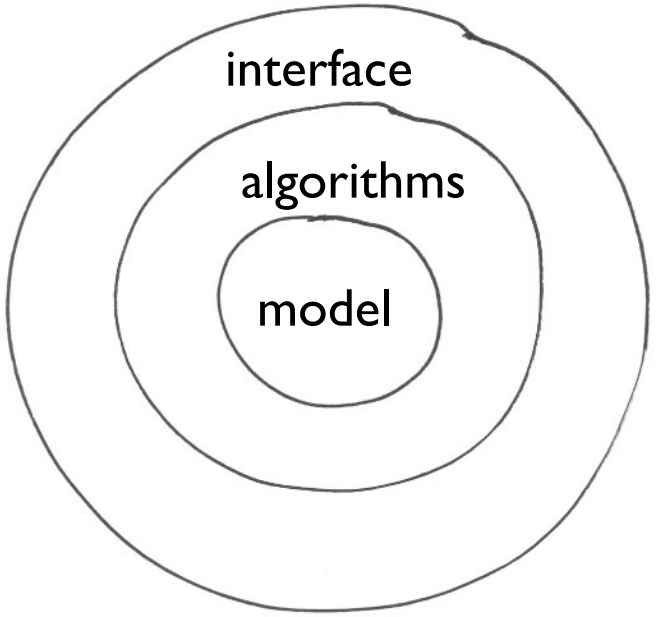
We can scale up the math!





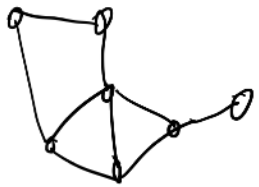
Courtesy : BMW

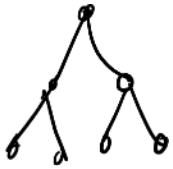
Models, algorithms and software



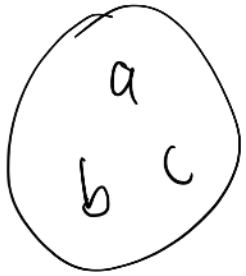
Bösendorfer mic'd for sampling

17





"Mathematics is the science of patterns!"



[a, b, c]

Associates both to patterns within mathematics itself, and to how we use them to model the world!

(Hardy, Steen)

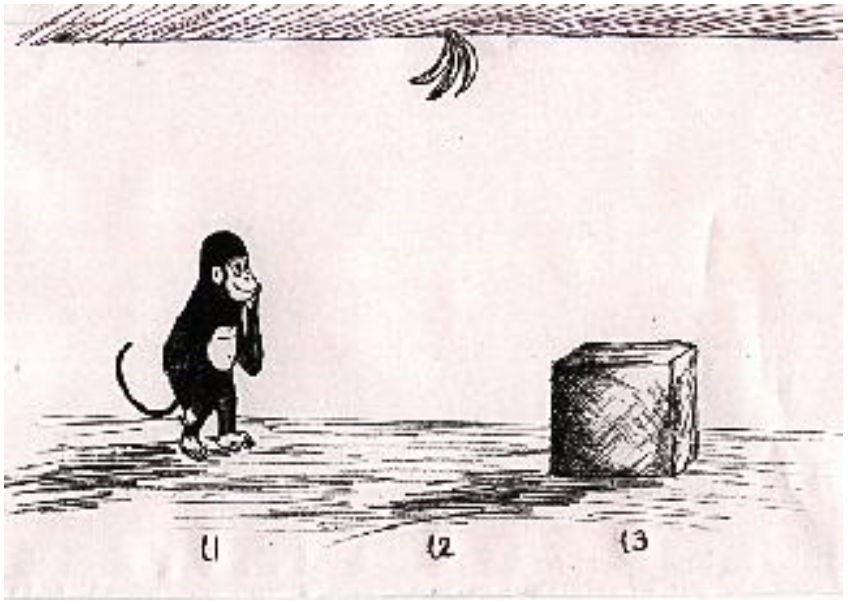
problem solving

Why problem solving?

It's the *variation*!

(Problems come in infinite variations - it will never be sufficient to learn a finite set of given methods)

How solve difficult problems with our limited capacity?



If you want to move this bookshelf to another wall you need to work in small steps!

Investigate the problem for
deeper understanding!

Explore different paths
towards a solution!

Always think, reflect
and simplify!

An important problem solving technique:
asking questions!

a question creates a subproblem!

heuristics: standard questions that are often good

An important method:
trying things out!

this is what all researchers use, but never document!

begin with something simple!

what have you learned about problem solving?

understand first!

try other options

break in smaller steps!

what have you learned about problem solving?

examples first!

simple first

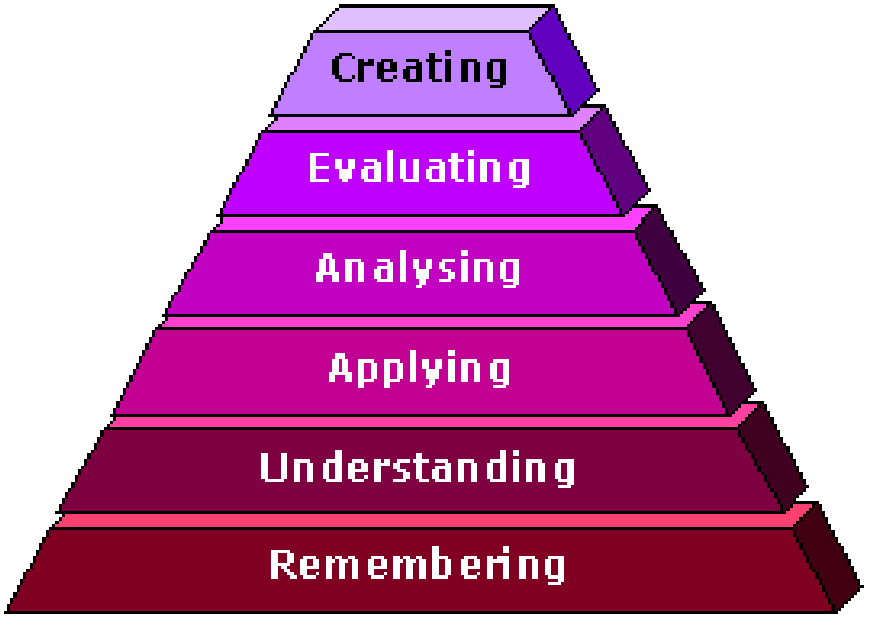
expect to make a discovery

(2017)

some final points

Bloom's taxonomy (1956, improved by Anderson et al 2001)

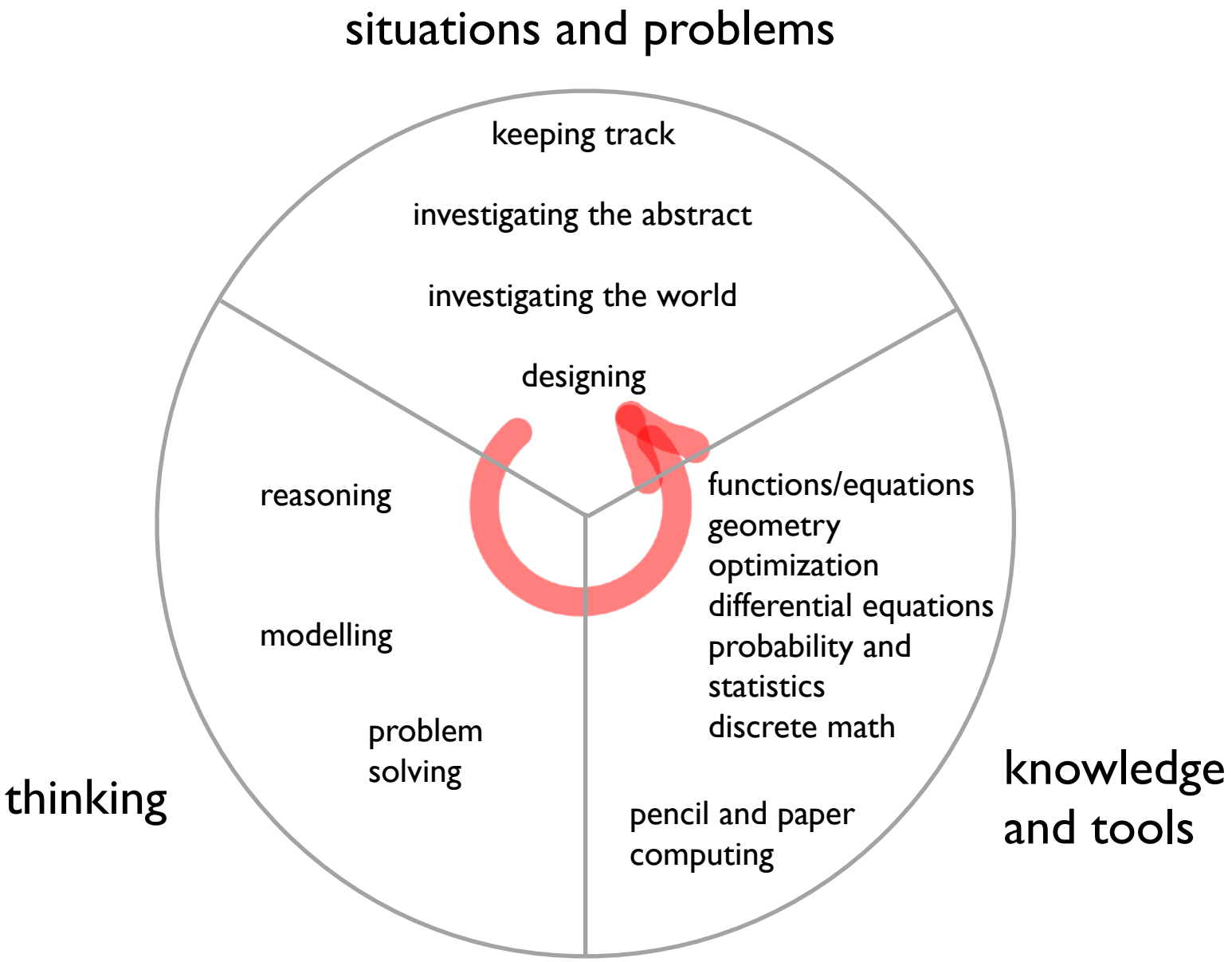
We try to move upwards in this hierarchy!



To do this you must take control of your own thinking!

Supplantive learning

http://skills4trainers.blogspot.se/2013/05/resistance-to-learning-and-learning.html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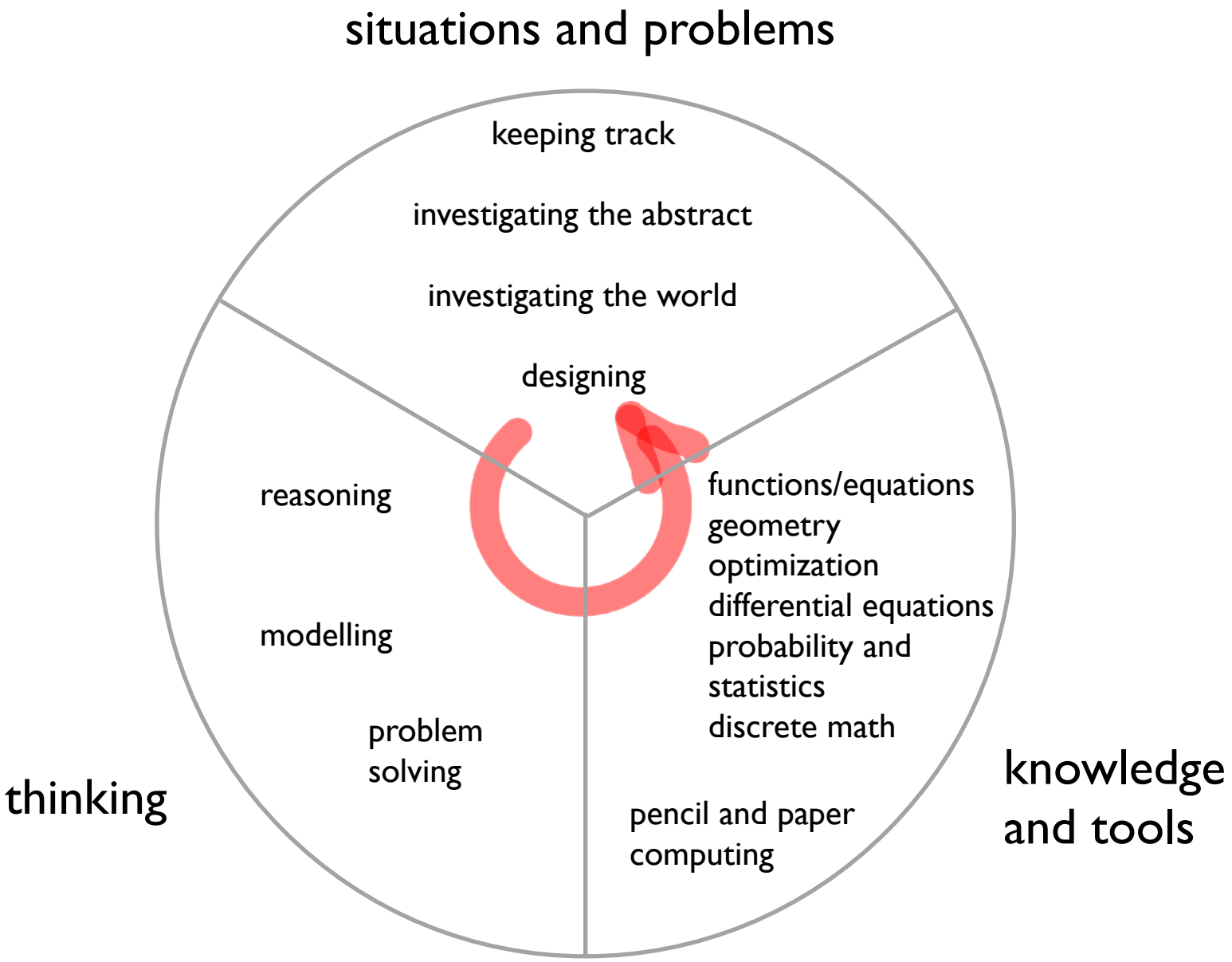
What's on your plate kids?
Salad & Veggies
Keep it Colourful
Protein
Carbohydrates
Nestlé Healthy Kids
AIS
Healthy Eating is as easy as 1,2,3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For you as a data scientist

Your ability to *think and work mathematically* is critical to your capacity as data scientists!

Many other "IT-people" do not have that.

Thank you!

END

"This is actually about knowing what you are able to do yourself and when you actually need help"

"The structure(s) of the course is an important part of what you learn"