

This is an explanation of:

multivariate probability distributions

marginal probability distributions

conditional probability

Bayes' theorem

events

- 11
- 01
- 00
- 11
- 11
- 00
- 10
- 01
- 11
- :

| $p(x_1, x_2)$ | | x_2 | | $p(x_1)$ marginal |
|---------------|---|-------|-----|--|
| | | 0 | 1 | |
| x_1 | 0 | 0.3 | 0.1 | 0.4 |
| | 1 | 0.2 | 0.4 | 0.6 |
| | | 0.5 | 0.5 | $p(x_2)$ marginal of $p(x_1, x_2)$ |

Now, what happens if we know $x_1 = 0$?
 How update with this new information?

Filtered
 events:

01
 00
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 ⋮

$p(x_1, x_2 | x_1 = 0)$

| | | x_2 | | |
|-------|---|-------|------|---|
| | | 0 | 1 | |
| x_1 | 0 | 0.75 | 0.25 | 1 |
| | 1 | 0 | 0 | 0 |
| | | 0.75 | 0.25 | |

$p(x_1 | x_1 = 0)$

from
 original
 table!

$$p(x_2 | x_1 = 0) = \frac{p(x_1 = 0, x_2)}{p(x_1 = 0)}$$

Same if $x_1=1$

$$P(x_2 | x_1=1) = \frac{P(x_1=1, x_2)}{P(x_1=1)}$$

| | |
|------|------|
| 0 | 0 |
| 0.33 | 0.67 |
| 0.33 | 0.67 |

So generally

$$P(x_2 | x_1) = \frac{P(x_1, x_2)}{P(x_1)}$$

| | |
|------|------|
| 0.75 | 0.25 |
| 0.33 | 0.67 |

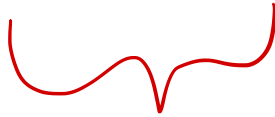
and similarly

$$P(x_1 | x_2) = \frac{P(x_1, x_2)}{P(x_2)}$$

\Rightarrow

marginal multiplied back to $p(x_2|x_1)$ to get the original $p(x_1, x_2)$!

$$p(x_1) p(x_2|x_1) = p(x_1, x_2) = p(x_2) p(x_1, x_2)$$



this part often
left out!

Bayes theorem!

Think in whole tables and
not just in single numbers!

$$p(x_1, x_2, x_3, x_4)$$

we can then just multiply
the marginals!

if independent

$$p(x_1, x_2, x_3, x_4) = p(x_1) p(x_2) p(x_3) p(x_4)$$

if pairwise dependent in chain (Markov)

$$p(x_1, x_2, x_3, x_4) = p(x_1) p(x_2|x_1) p(x_3|x_2) p(x_4|x_3)$$

(follows from repeated application of Bayes theorem and the Markov property)