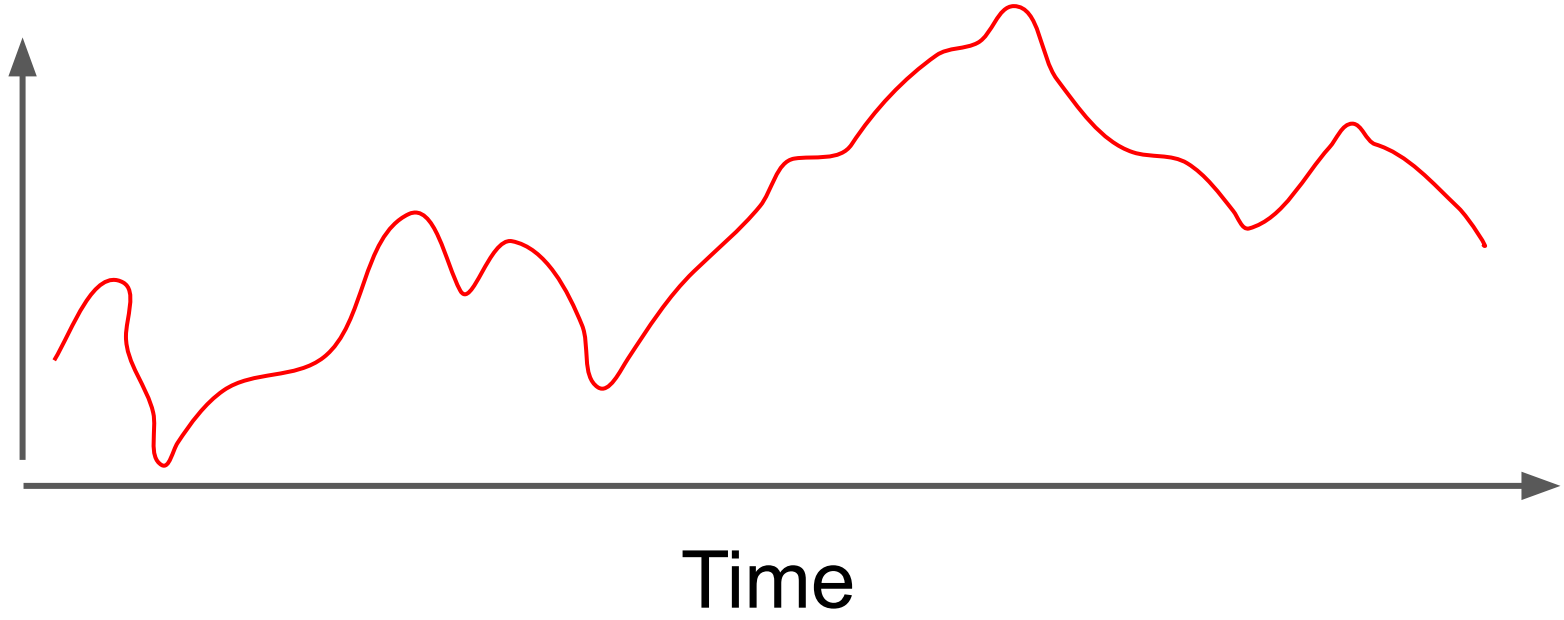
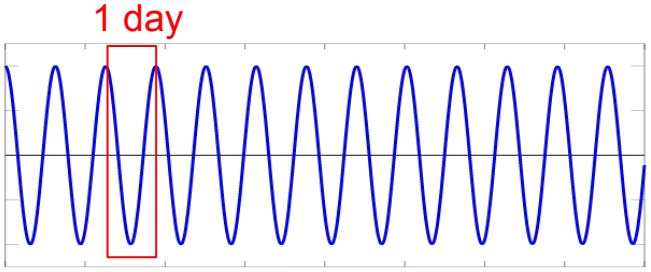
Dynamic systems

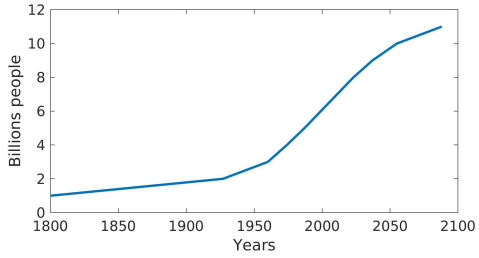


1998 2019 (wiki/google)

Data

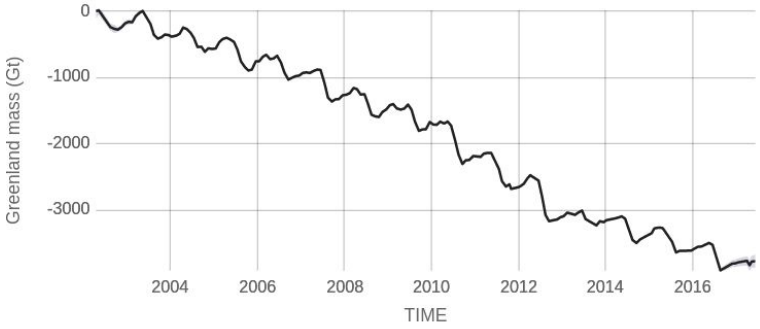


Air temperature



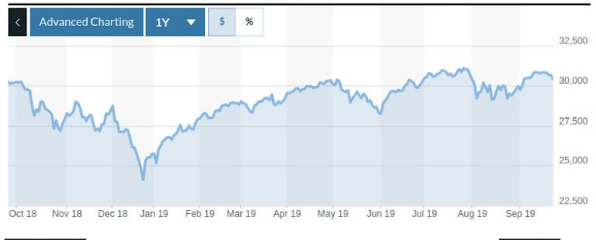
Human population

Greenland ice cap



Source: climate.nasa.gov

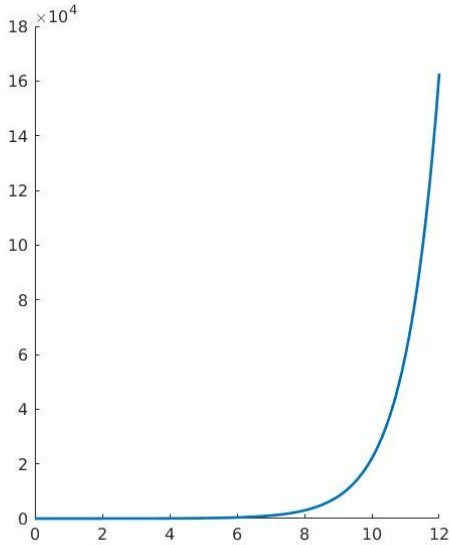
Stock market (marketwatch.com)



Derivative

- notation
- “Geometric” approach (slope of the tangent)
- Infinitesimal variation
- Daily life derivative ?

$$y' = ky$$

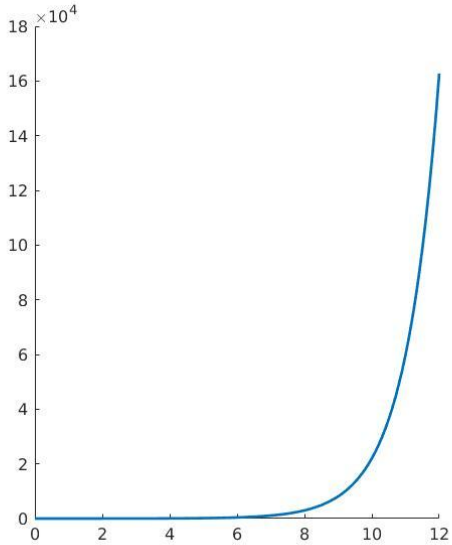


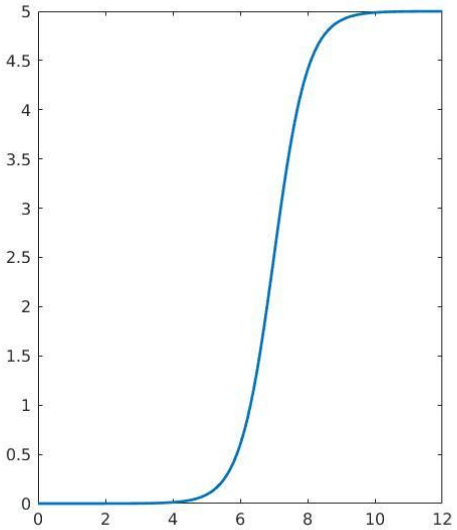
Population growth

$$y' = ky$$

$$\Downarrow$$

$$y' = ky(1 - \frac{y}{L})$$





Differential equations

- Links function to its derivative
- Often used in dynamic systems
- Basically two options to solve it

System of differential equations

- Whales w(t) and krill k(t)

$$\begin{cases} k' = (a - bw)k \\ w' = (-m + nk)w \end{cases}$$

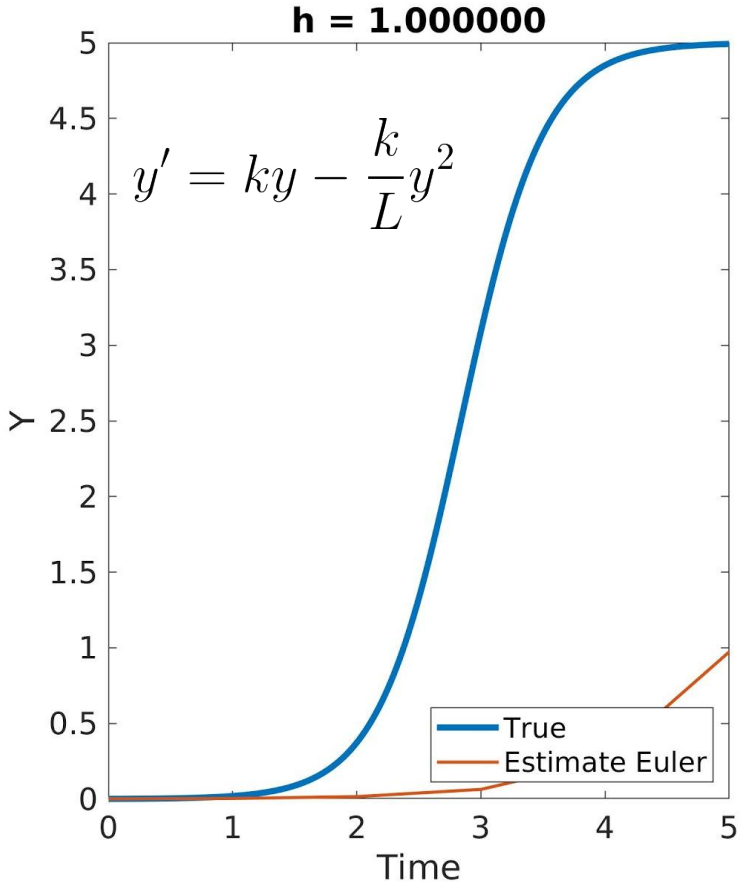
Explain

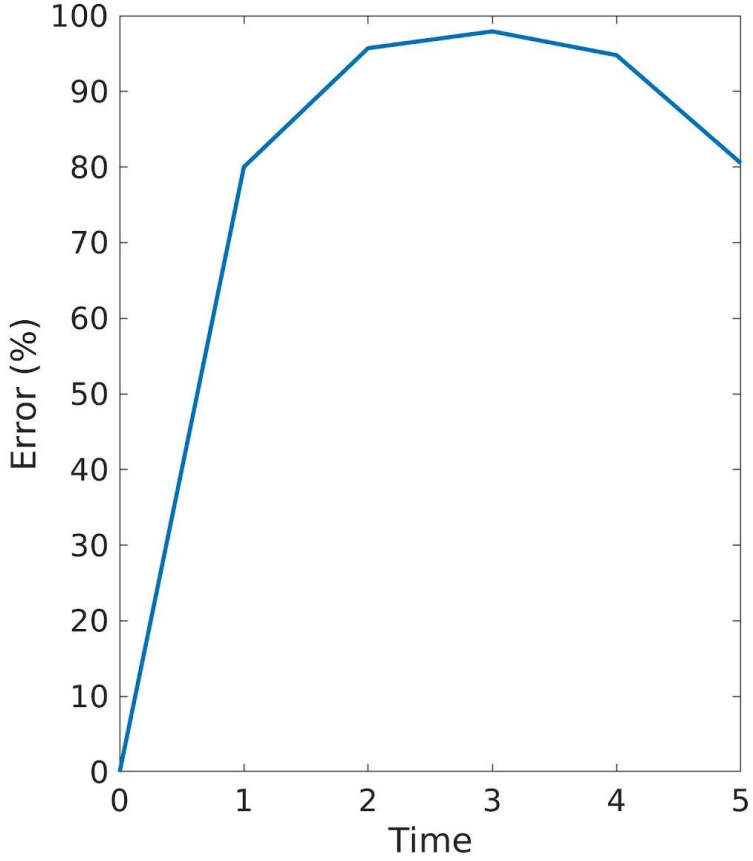
Solve a differential equation

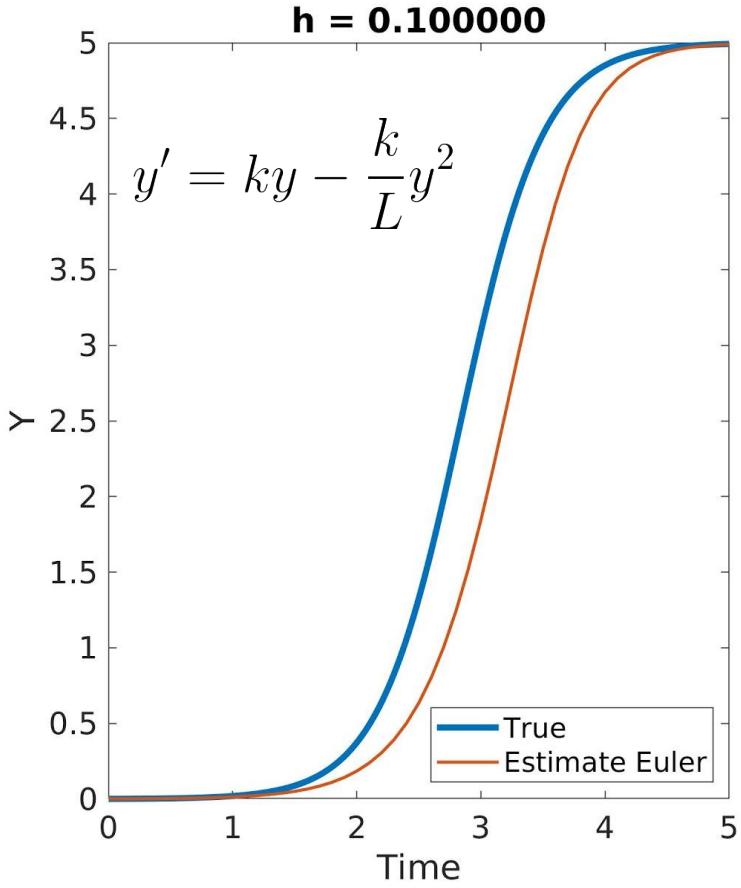
- Analytical solution
- Simulation → e.g. Euler method - iterative

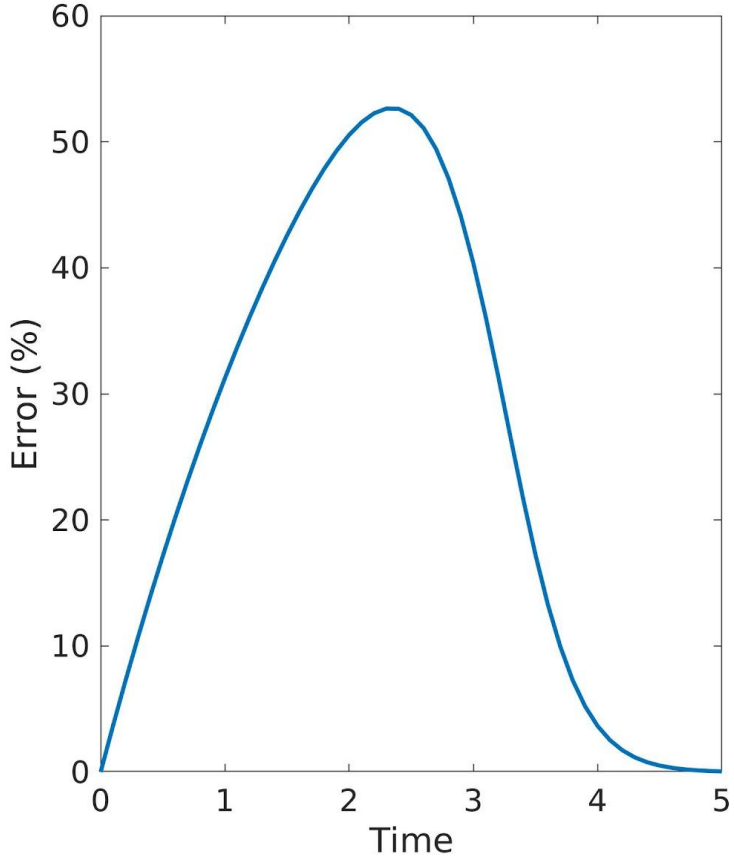
$$k(t+h) = k(t) + k'(t)h$$
$$w(t+h) = w(t) + w'(t)h$$

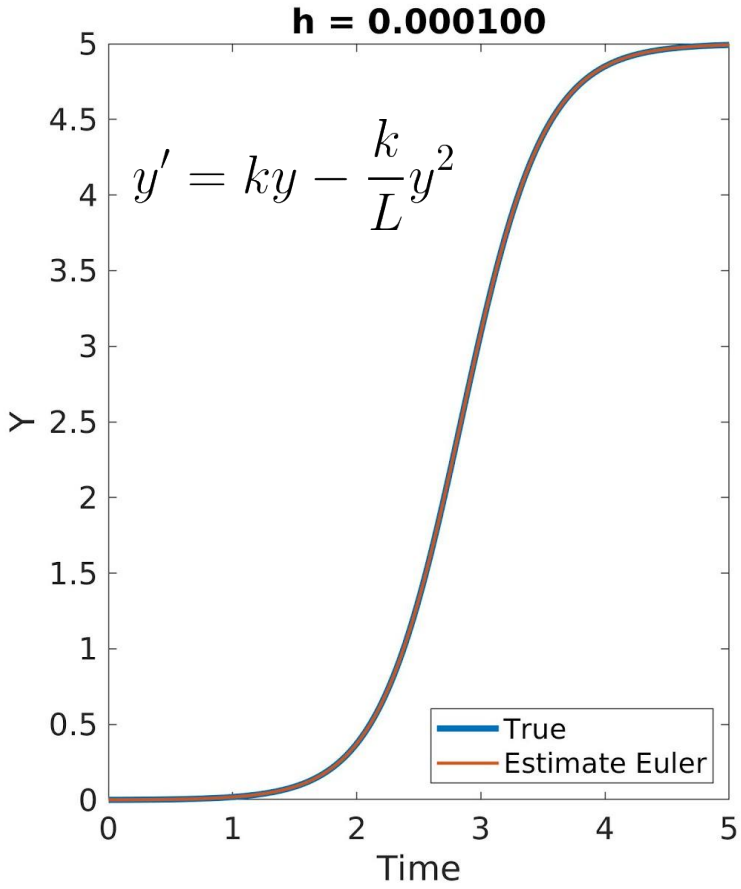
- Needs?

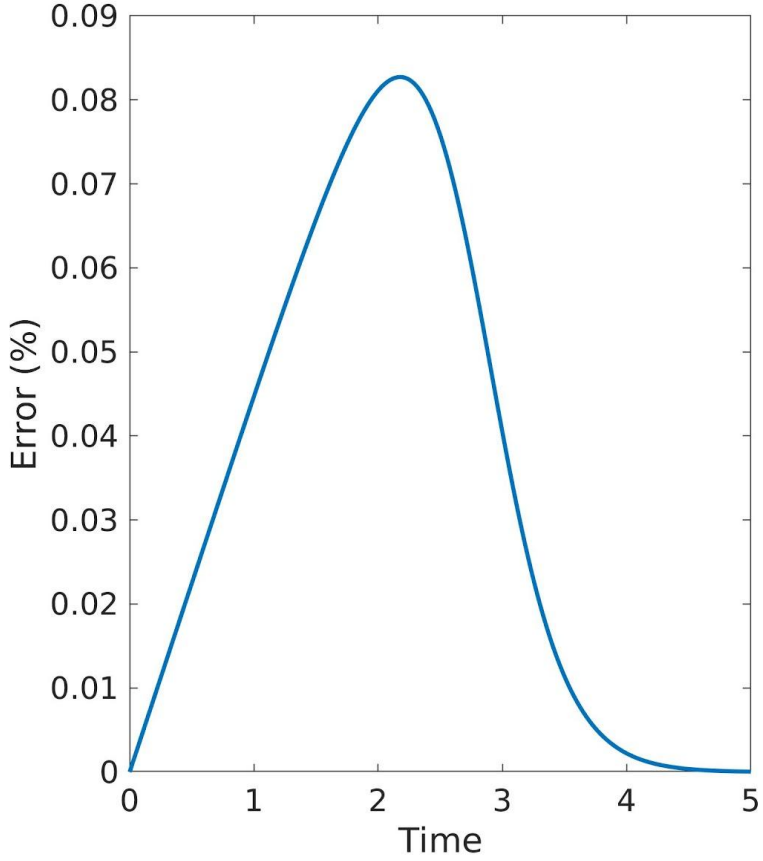

h = 1.000000
$y' = ky - \frac{k}{L}y^2$
Y
Time
True
Estimate Euler
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
1
2
3
4
5

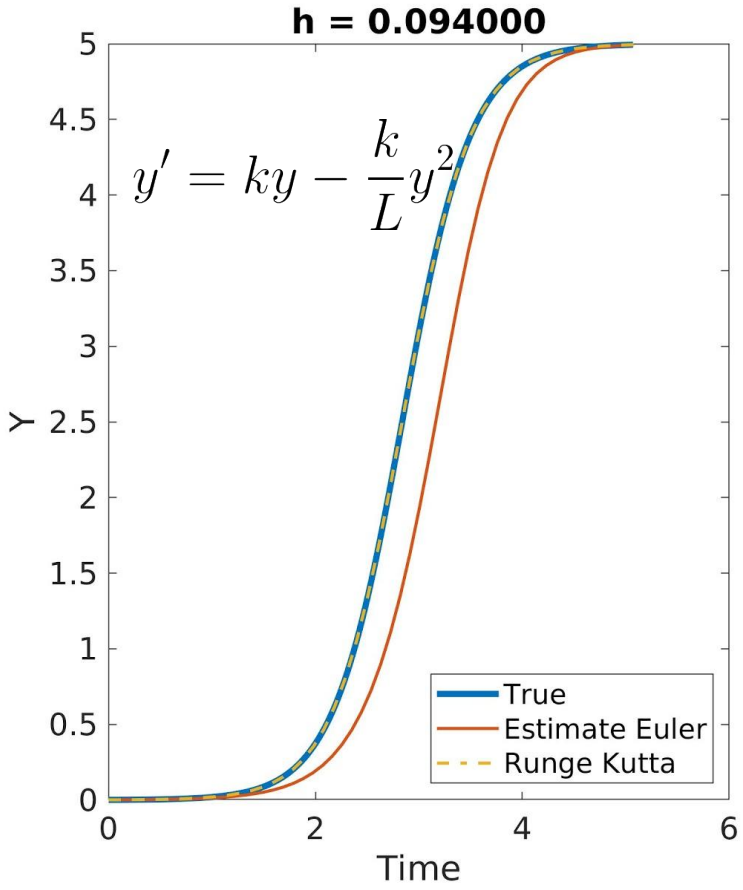

Error (%)
Time
0
10
20
30
40
50
60
70
80
90
100
0
1
2
3
4
5

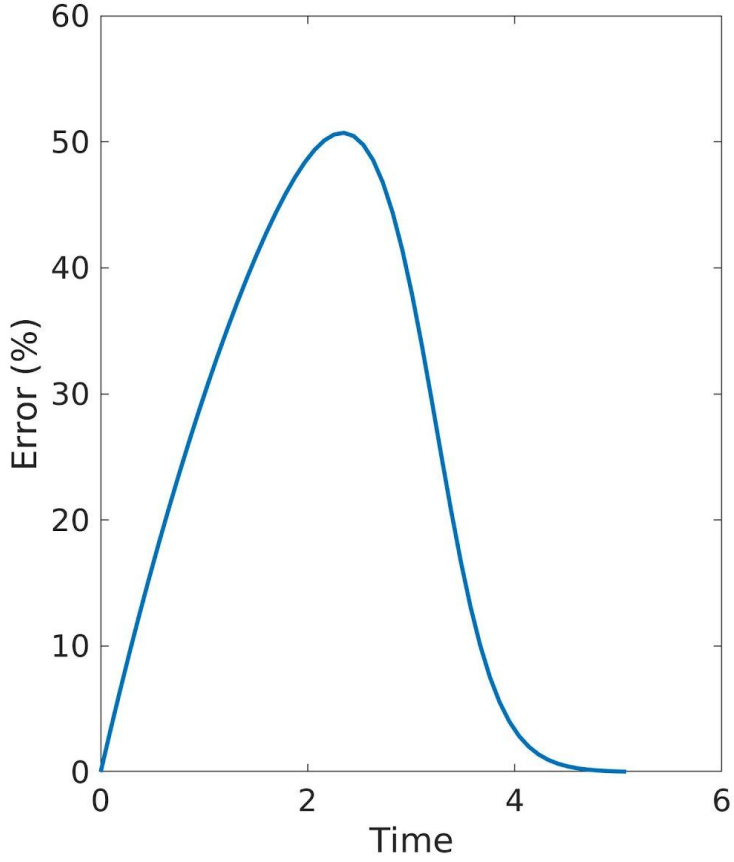

h = 0.100000
$y' = ky - \frac{k}{L}y^2$
Y
Time
True
Estimate Euler
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
1
2
3
4
5


Error (%)
Time
0
10
20
30
40
50
60
0
1
2
3
4
5


h = 0.000100
$y' = ky - \frac{k}{L}y^2$
Y
Time
True
Estimate Euler
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
1
2
3
4
5


Error (%)
Time
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0
1
2
3
4
5


h = 0.094000
$y' = ky - \frac{k}{L}y^2$
Y
Time
True
Estimate Euler
Runge Kutta
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
2
4
6


Error (%)
Time
0
10
20
30
40
50
60
0
2
4
6

System of differential equations

- Whales w(t) and krill k(t) → Stable points

$$\begin{cases} k' = (a - bw)k \\ w' = (-m + nk)w \end{cases}$$

Integration

- y’ → y

$$\int_a^b f(x)dx$$