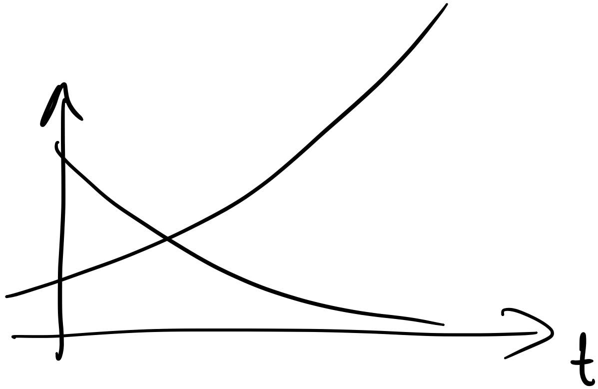
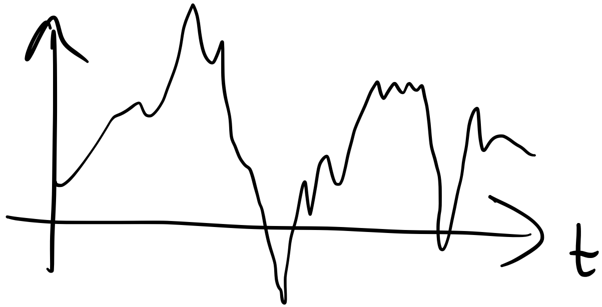
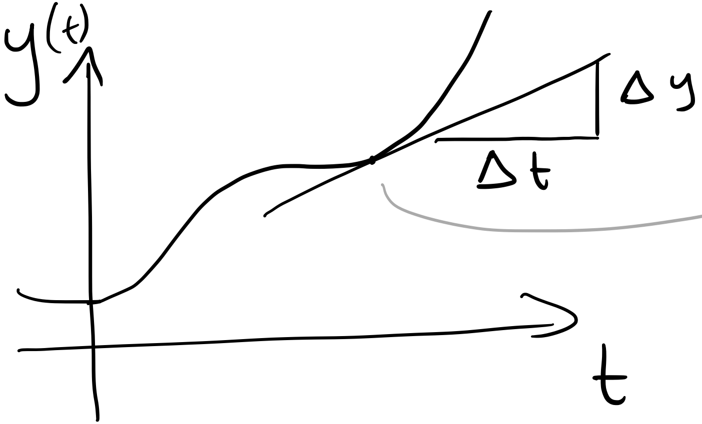
Dynamic systems intro

change over time







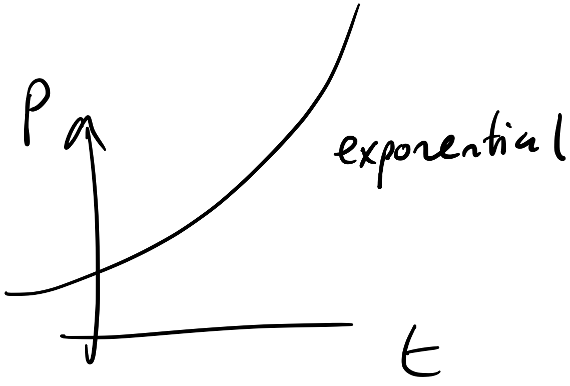
the derivative in this point!

$$\text{rate of change} = \text{derivative} = \frac{\Delta y}{\Delta t}$$

The derivative is the key to the mechanism of change!

eg speed

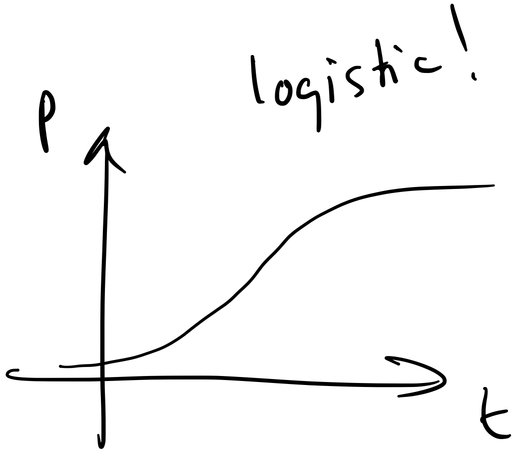
population growth

$p' = c \cdot p$ $\Longrightarrow$ 

simple proportional mechanism!

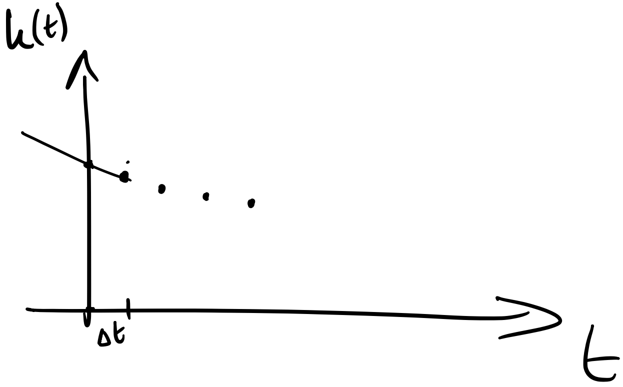
p(t) is the accumulated effect

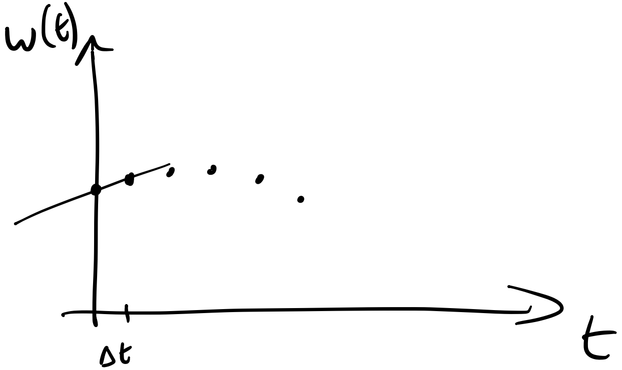
$$\underbrace{p' = r\,p\,(M-p)}_{\text{another mechanism for the rate of change}} \implies$$



krill and whales

$$\begin{cases} k' = (a - bw)\,k \\ w' = (-m + nk)\,w \end{cases}$$



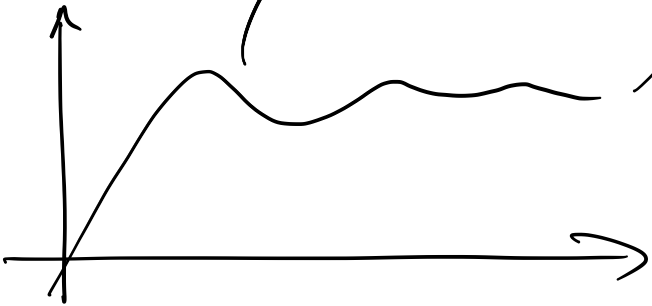


a, b, m, k constants

a system of <u>differential equations</u>

Differential equations are often not difficult to formulate

easy to simulate!



easy to determine final value in the long run!
(if it exists)

how?

Some can be solved analytically

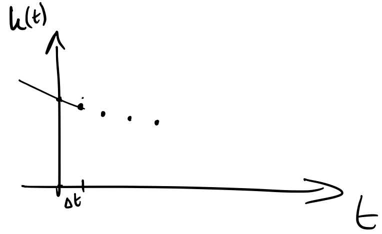
$$p' = r p (M - p)$$

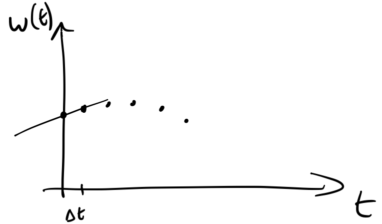
$$\Rightarrow p = \frac{M}{1 + c e^{-rt}}$$

Nice when it works!

linear yes, non-linear most often no

$$\begin{cases} k' = (a - bw)\,k \\ w' = (-m + nk)\,w \end{cases}$$





$$\begin{cases} k(t+\Delta t) = k(t) + \underline{k'(t)} \cdot \Delta t \\ w(t+\Delta t) = w(t) + \underline{w'(t)} \cdot \Delta t \end{cases}$$

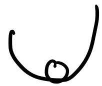
for simulation!

Eulers method

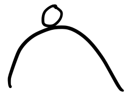
set derivatives to 0!

$$\begin{cases} 0 = (a - bw)k \\ 0 = (-m + nk)w \end{cases}$$

solve for equilibrium points!



stable equilibrium



unstable equilibrium

Moving from f' to f is called

<u>integration.</u>

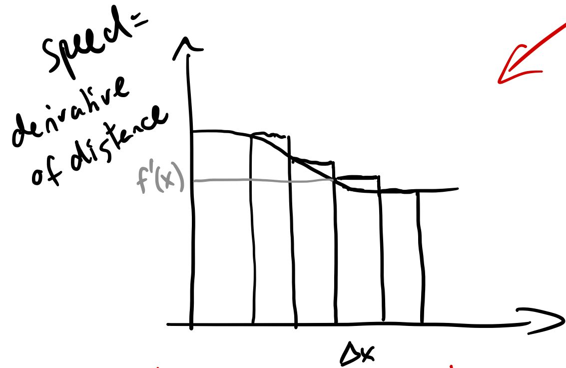
in every step we add the area of one more rectangle!

$$f(x+\Delta x) = f(x) + \underline{f'(x)\cdot\Delta x}$$

One way to see it

accumulated effect of speed

Another way to see it



We accumulate the area under the curve