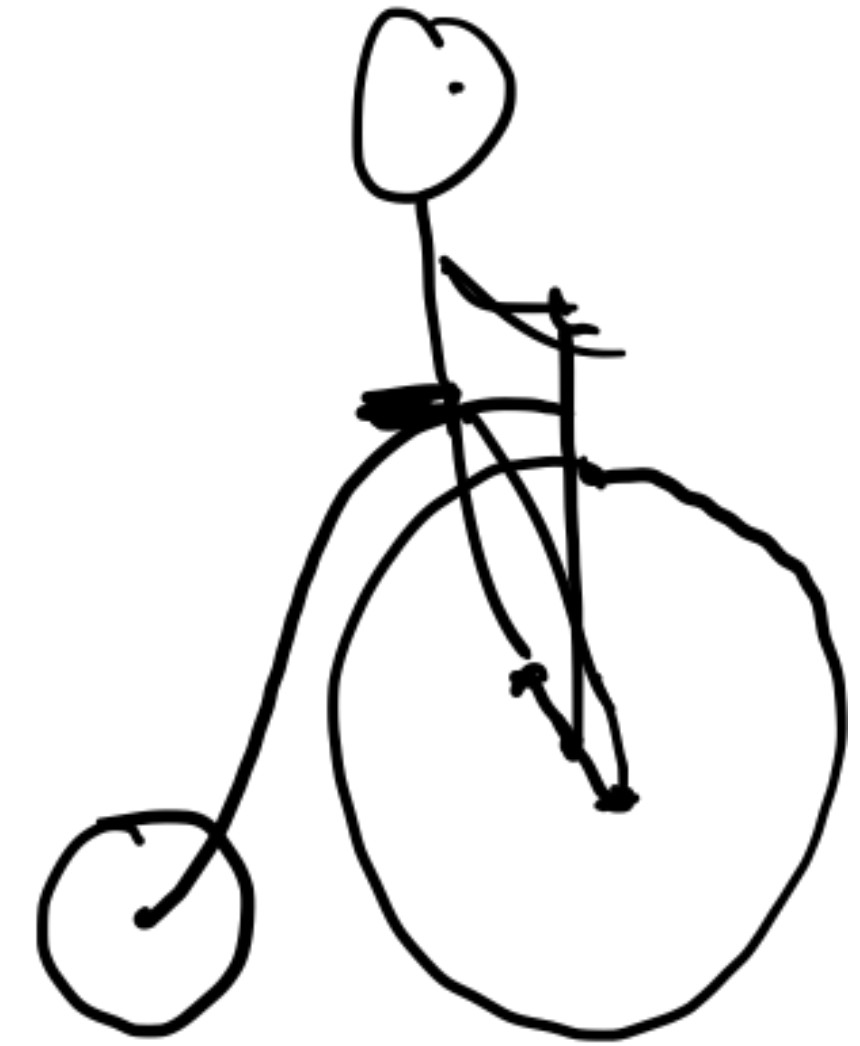
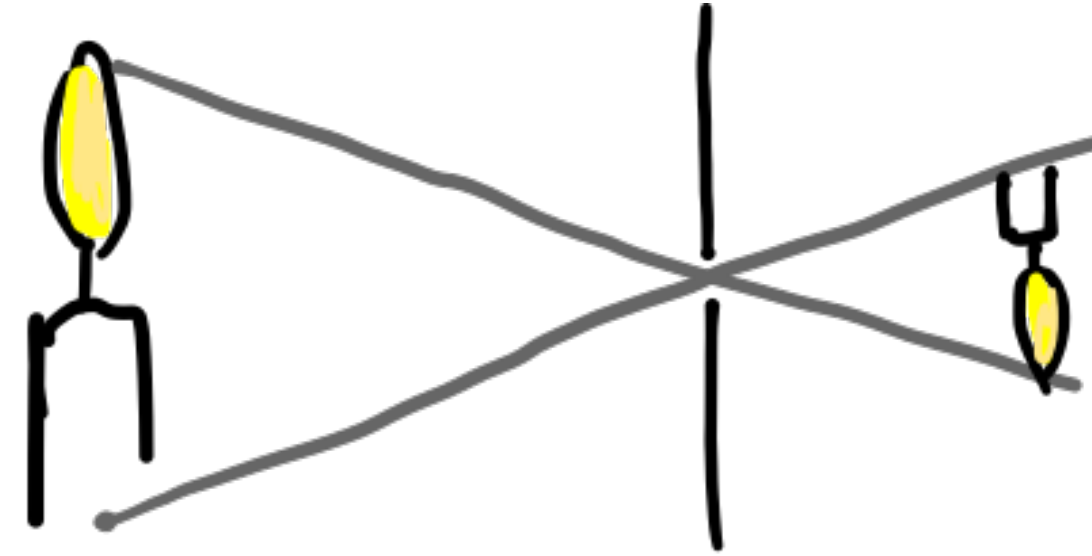
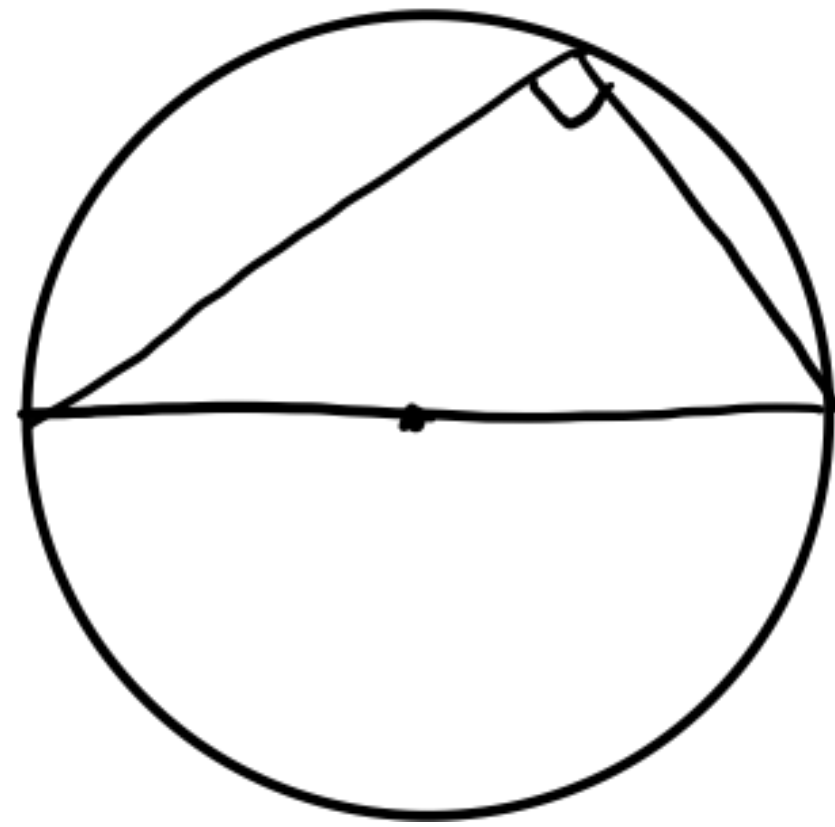
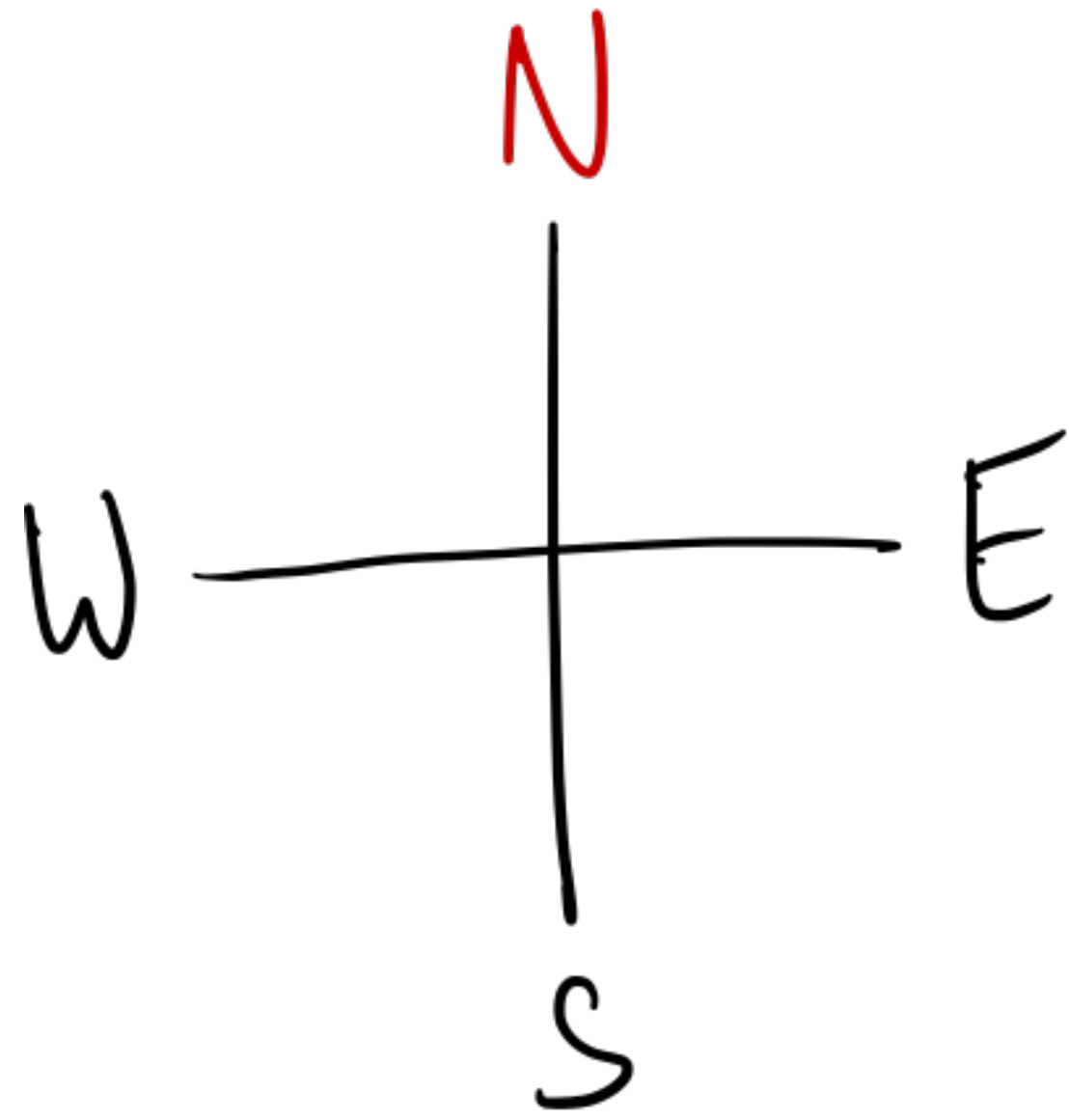


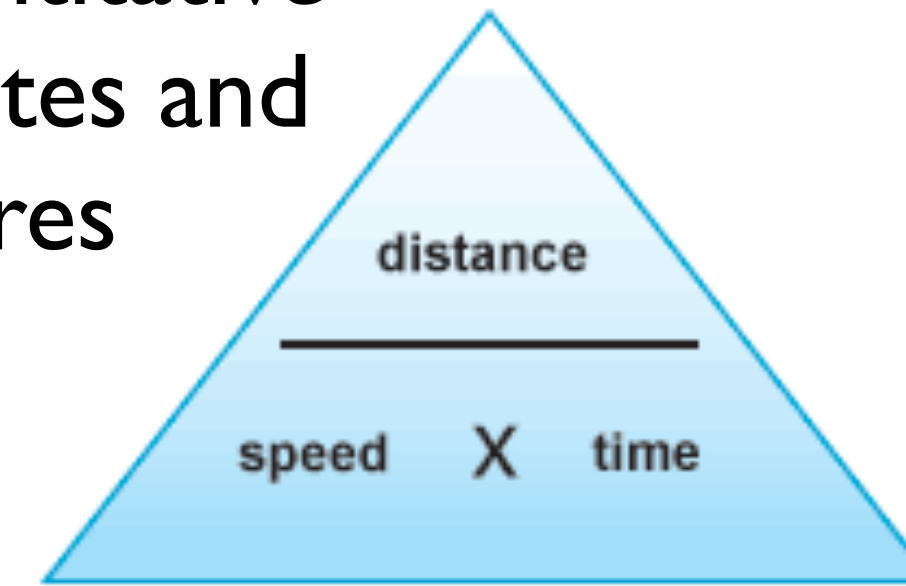
Where and how is mathematical thinking used?



# Keeping track

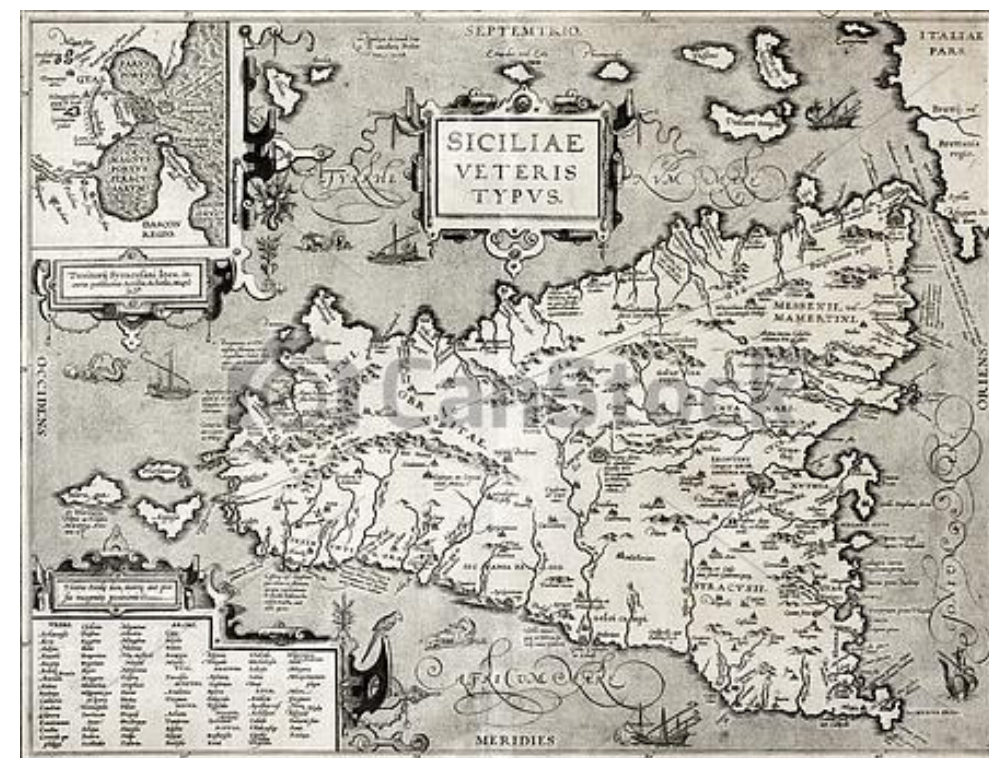


different kinds  
of quantitative  
attributes and  
measures

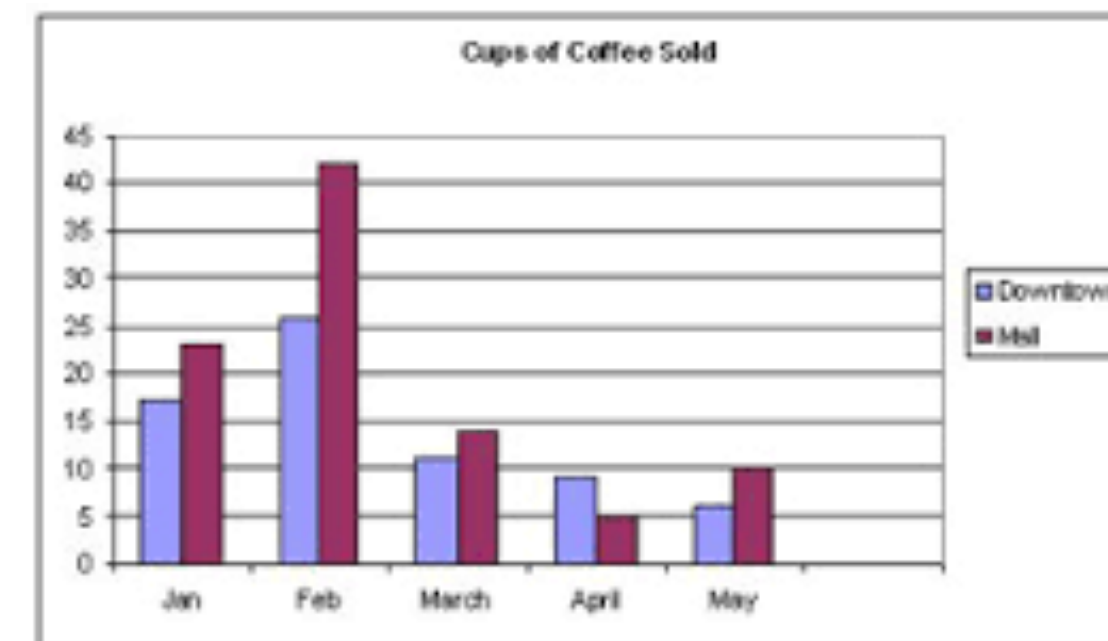


*keeping track is  
centered around  
practical needs*

*“numbers  
around us”*



© Can Stock Photo - csp10798633



*usually nothing  
interesting is expected  
- it can be useful but  
we just keep track*

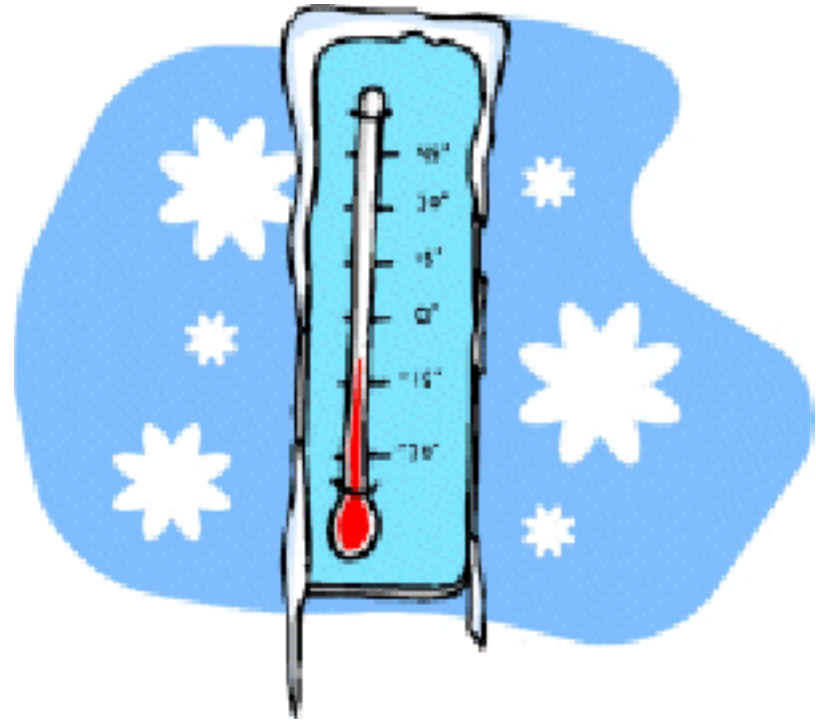
*some of this is too basic for this course!*

Keeping track of quantities makes it possible to use numbers in many situations...

(qualitative and quantitative understanding)

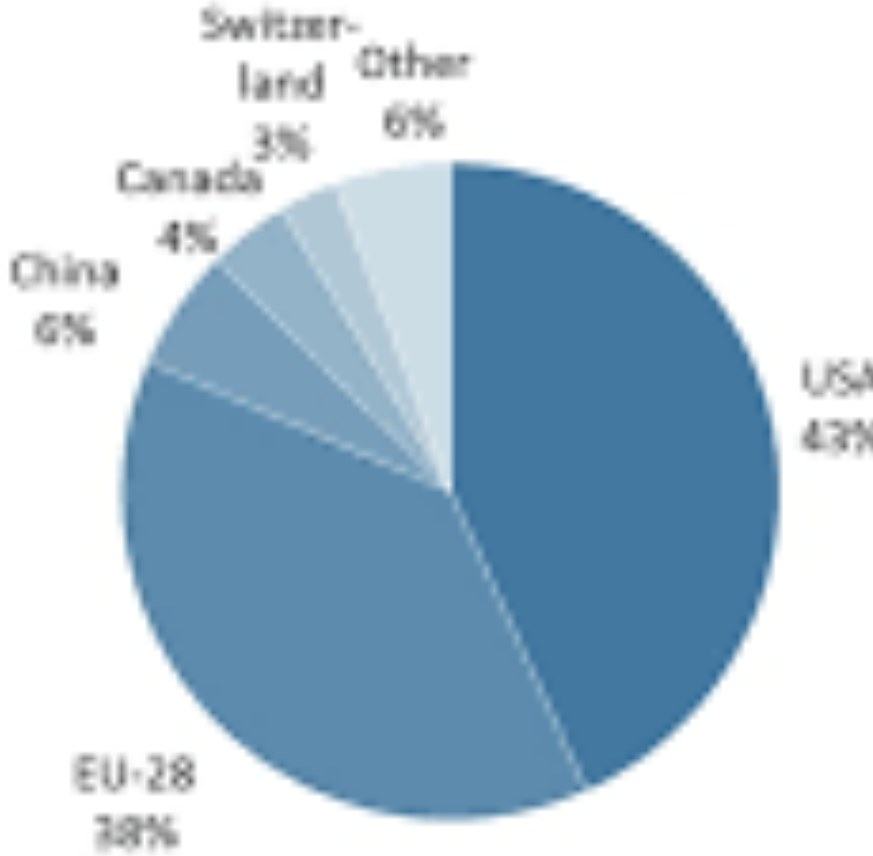
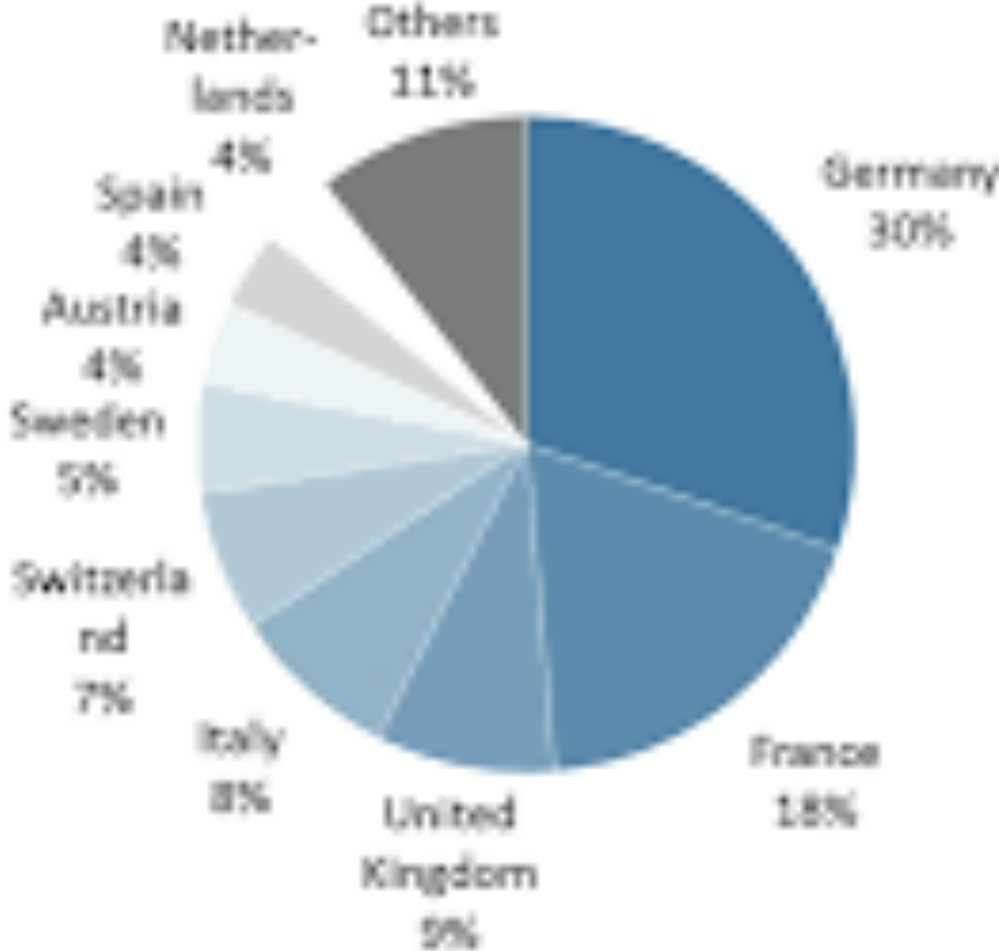
(decisions)

(how much do we need?)



Europe: Distribution of retail sales 2014  
Source: FIBL-AMI survey 2016

World: distribution of retail sales by single market 2014  
Source: FIBL-AMI survey 2016

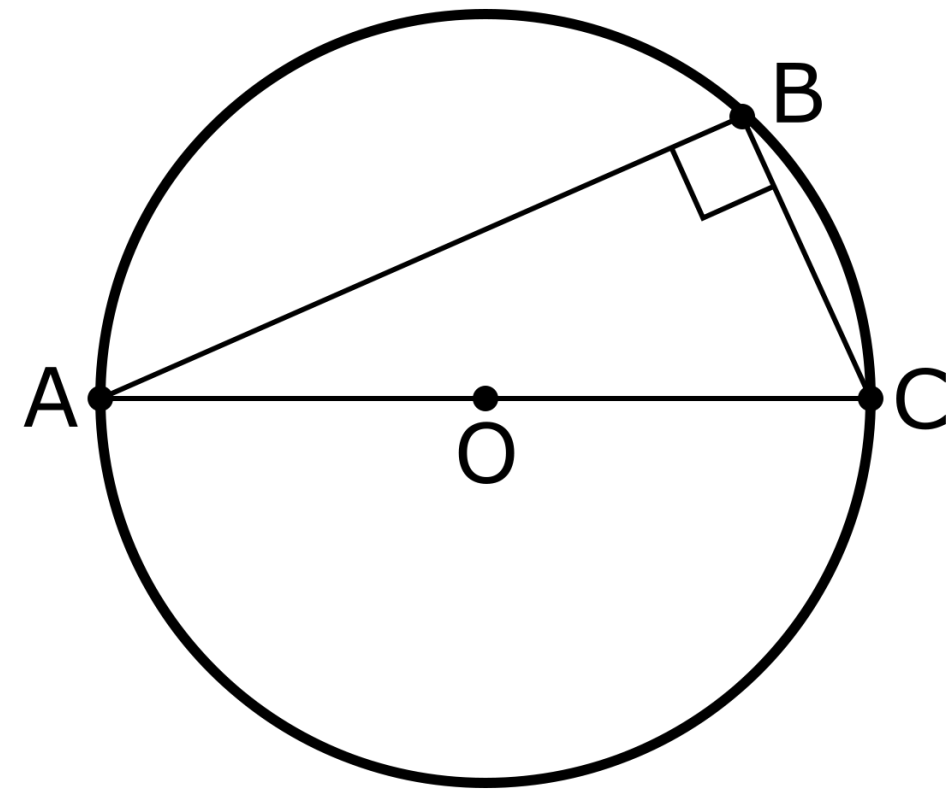


“It’s really cold today so I take my warmest coat”

“Fifteen litres of water will be enough”

“Sweden is a small market for organic foods, but large in proportion to its population”

# Investigating the abstract



Mathematics itself  
(numbers, shapes,...)



Particular abstract problems  
(games, strategies, algorithms,...)

Why not just *understanding*  
the abstract?

To investigate is to carefully examine and draw conclusions.

This is a way to create knowledge.

*Investigating is sustainable learning! Just learning other people's solutions is not enough!*

*Everyone can learn to investigate, but it requires practice! You practice and extend your natural ability to think*



*An investigation often begins with a question - and continues with other questions that you formulate along the way*

*Investigating requires hard work and patience!*

*“Seek and you shall find!”*

consider varied  
realistic problems

Situations  
and problems

practice entire  
process

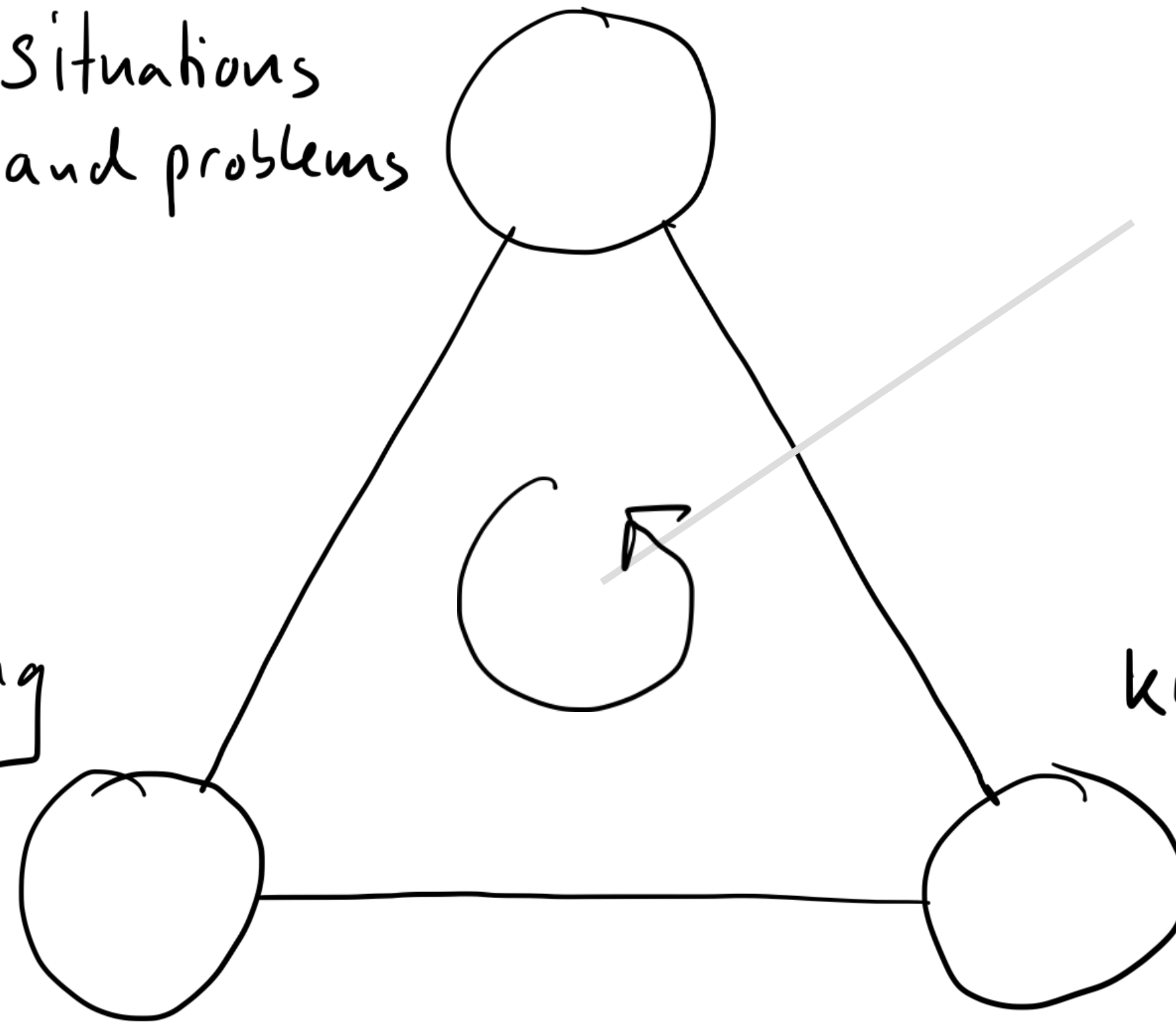
thinking

knowledge

practice your  
thinking

try to discover  
own knowledge

learn knowledge  
found by others



What comes first?

**Why do we need to know programming?**

**So much code has already been written!**

What do we mostly do in math education?

**How can I possibly come up with anything new in math with so many smart people working for so long?**

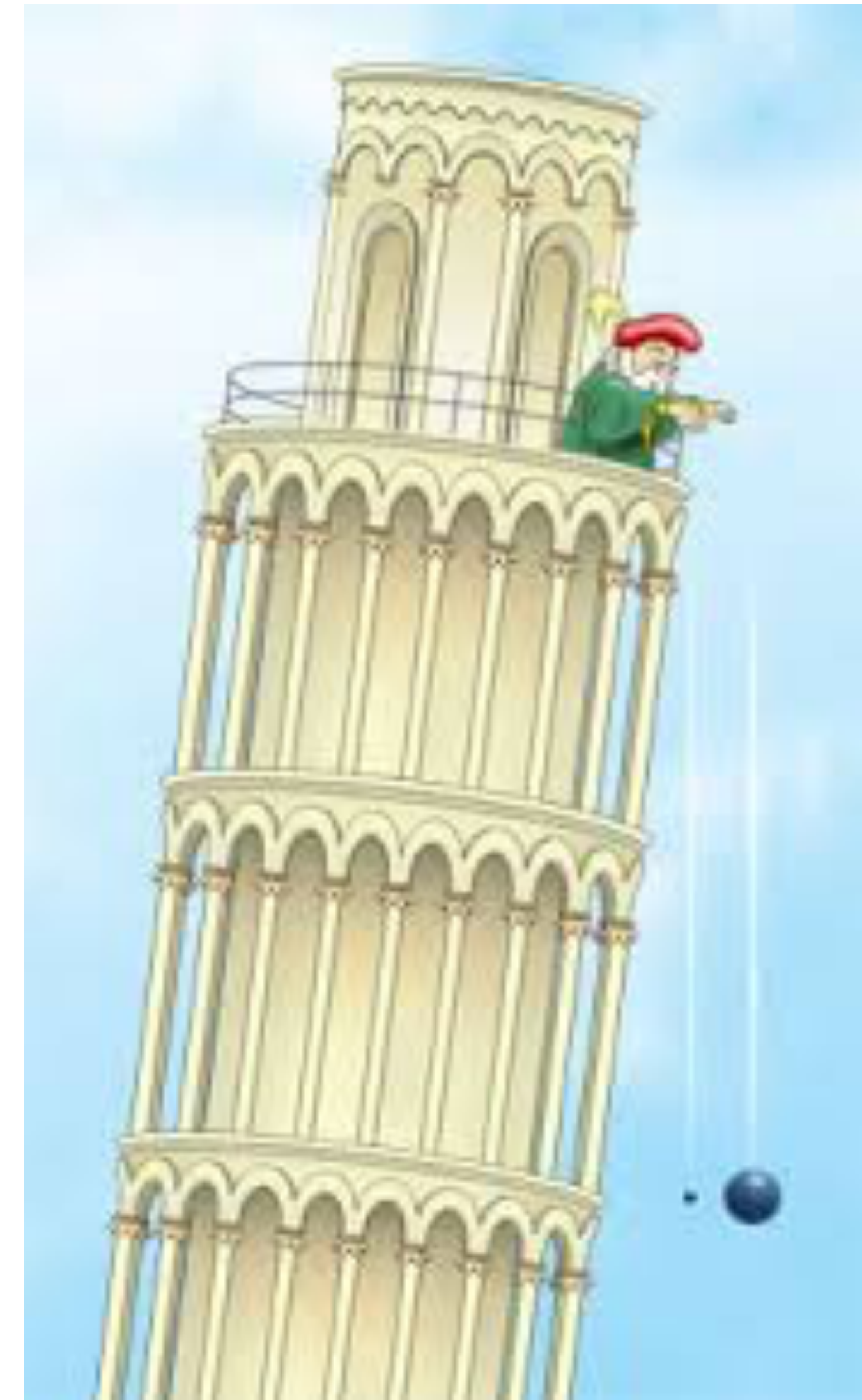
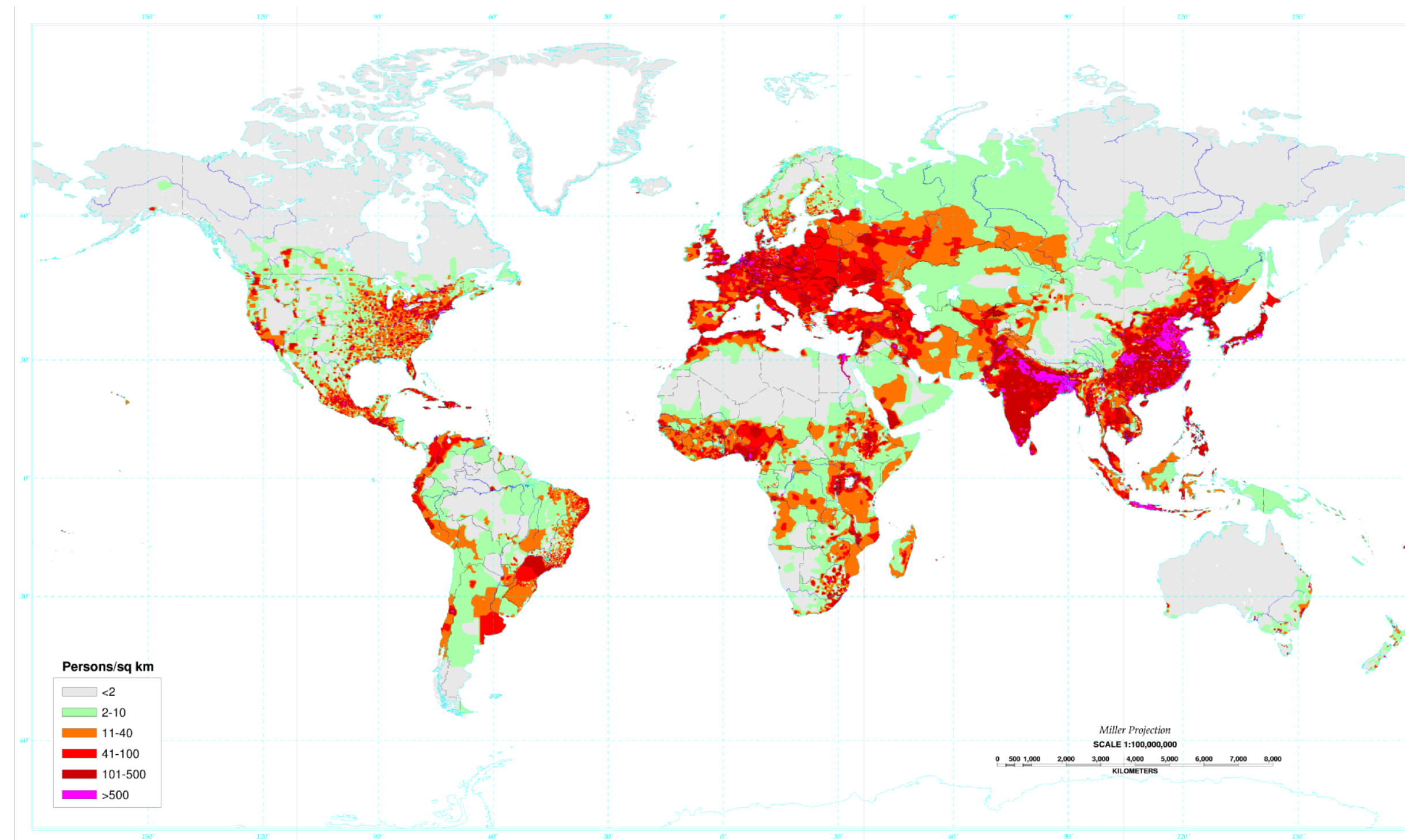
*“we want students to be able to use the mathematical knowledge they have learned in new situations”*  
(is this a good way to think?)



investigating the world

# Collecting data and empirical modelling

We can learn first hand about the world by collecting data, doing experiments, and drawing conclusions from these



# Mechanistic modelling



Why does the moon follow us?

WHY DOES THE MOON FOLLOW US?

If we have some knowledge, we can find out more by just thinking and drawing conclusions

this is all deductive!

## "Primitive" Weather Forecasting Equations

$p = \rho R T$  Ideal Gas Law (Equation of State)

$\bar{a}_h = \sum \left( \frac{\bar{F}_h}{m} \right)$  Newton's Second Law of Motion

$\Delta p = -\rho g \Delta z$   
 $(PGA)_v = g$   
 $\bar{a}_v = \sum \left( \frac{\bar{F}_v}{m} \right) = (\bar{P}\bar{G}\bar{A})_v - \bar{g}$

Hydrostatic Law (Obtained from the Equation of Vertical Motion)

$\Delta T = \Delta q / c_p + (1/\rho) \Delta p$  First Law of Thermodynamics

$(1/\rho) \Delta \rho / \Delta t = -DIV$

Conservation of Mass Applied to the Atmosphere (Equation of Continuity)

$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \omega \left( \frac{\partial T}{\partial p} + \frac{RT}{pc_p} \right) = \frac{J}{c_p}$      $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$      $0 = -\frac{\partial \phi}{\partial p} - \frac{RT}{p}$

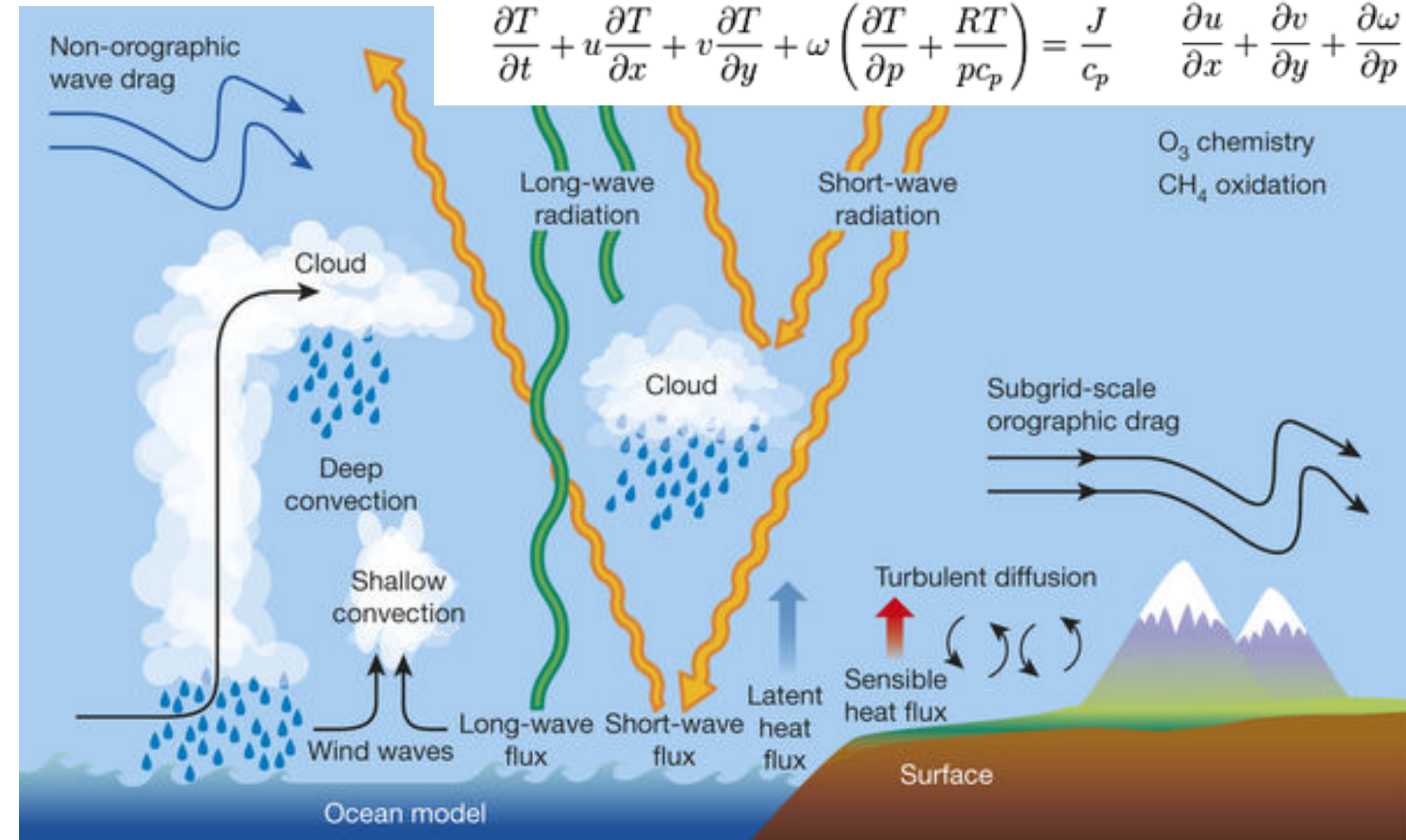
Zonal wind:  
 $\frac{\partial u}{\partial t} = \eta v - \frac{\partial \Phi}{\partial x} - c_p \theta \frac{\partial \pi}{\partial x} - z \frac{\partial u}{\partial \sigma} - \frac{\partial (u^2 + v^2)}{\partial x}$

Meridional wind:  
 $\frac{\partial v}{\partial t} = -\eta u - \frac{\partial \Phi}{\partial y} - c_p \theta \frac{\partial \pi}{\partial y} - z \frac{\partial v}{\partial \sigma} - \frac{\partial (u^2 + v^2)}{\partial y}$

Temperature:  
 $\frac{\delta T}{\delta t} = \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z}$

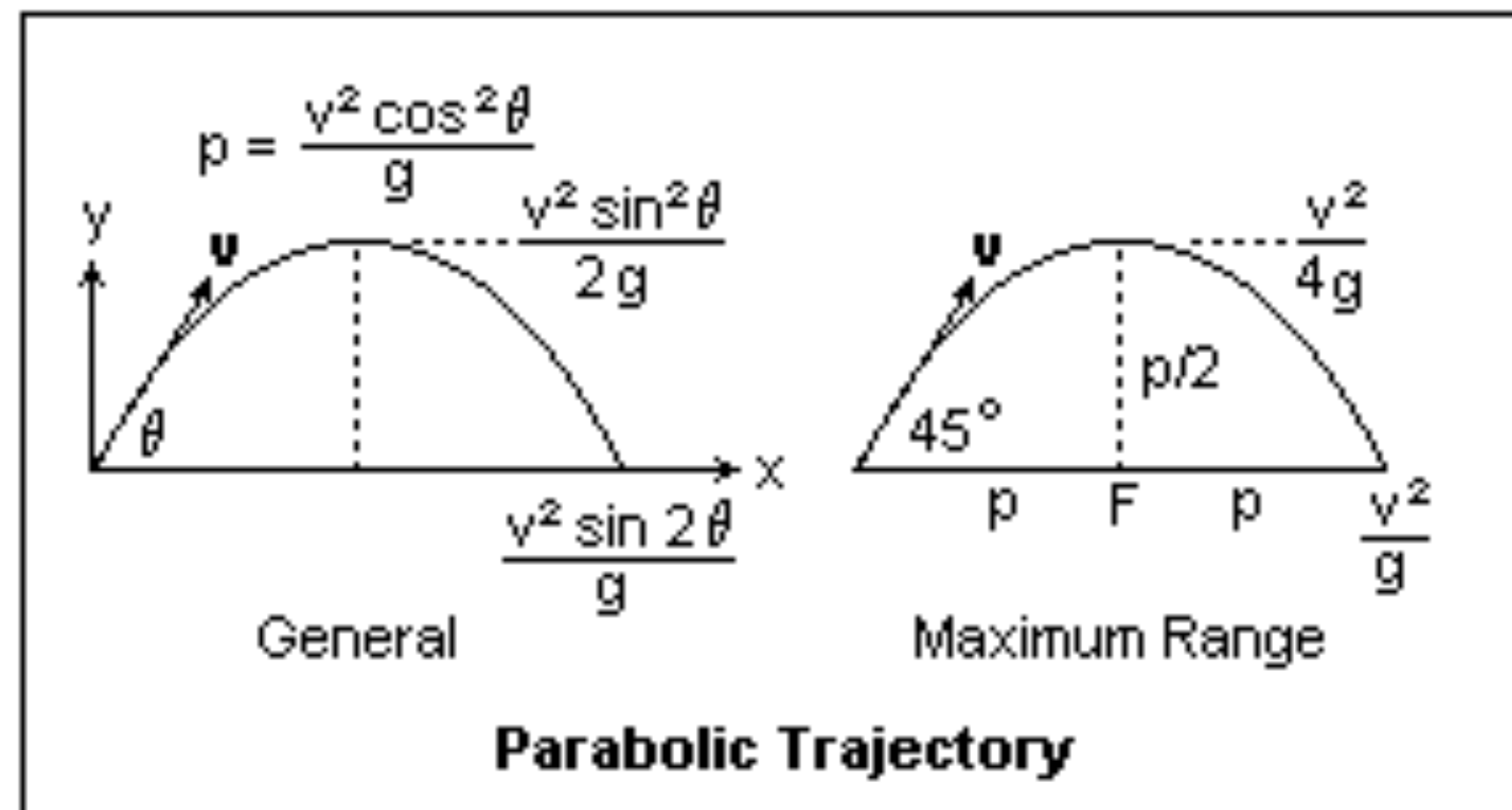
Precipitable water:  
 $\frac{\delta W}{\delta t} = u \frac{\partial W}{\partial x} + v \frac{\partial W}{\partial y} + w \frac{\partial W}{\partial z}$

Pressure thickness:  
 $\frac{\partial}{\partial t} \frac{\partial p}{\partial \sigma} = u \frac{\partial}{\partial x} \frac{\partial p}{\partial \sigma} + v \frac{\partial}{\partial y} \frac{\partial p}{\partial \sigma} + w \frac{\partial}{\partial z} \frac{\partial p}{\partial \sigma}$

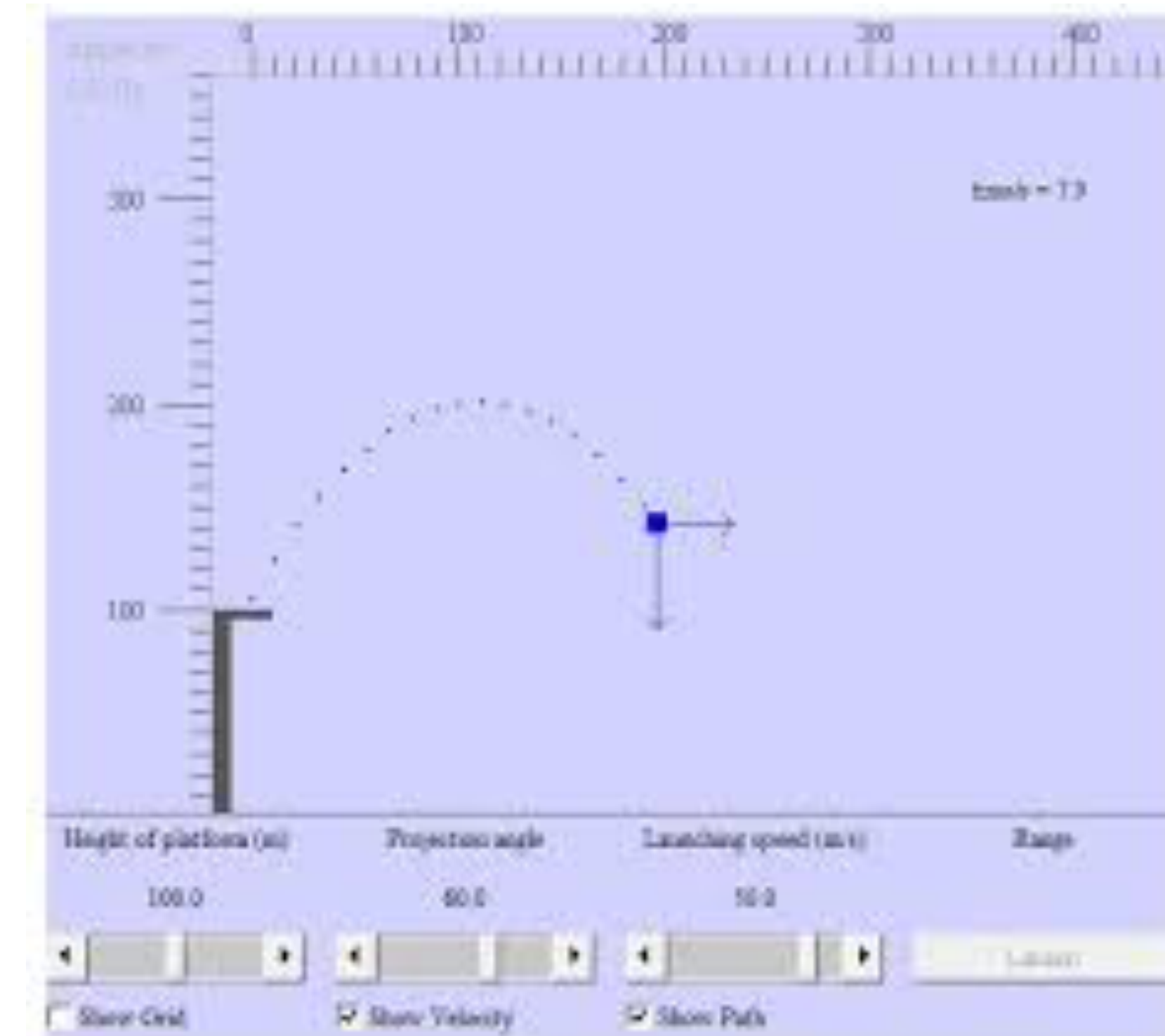


If we know well how things work we can create models that simulate reality in the computer and learn even more!

Two fundamentally different ways to do calculations about the real world



Simple direct calculation for easy things



Step by step simulation is feasible for much more complicated systems!

Qualitative understanding is more important than full quantitative understanding

*“there are atoms and molecules”*

*“the apple falls to the ground”*

*“this culture of borrowing is not healthy”*

*“basically, the population increases in proportion to its size”*

*In research, quantitative investigation can lead to significant qualitative understanding*

*When you do calculations, never lose track of your understanding of the corresponding reality!*

Many aspects of nature can be explained  
surprisingly well with mathematics!

*Why?*

*What can, and what cannot, be explained  
well with math?*

designing

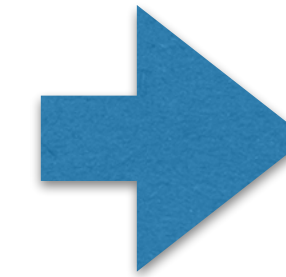
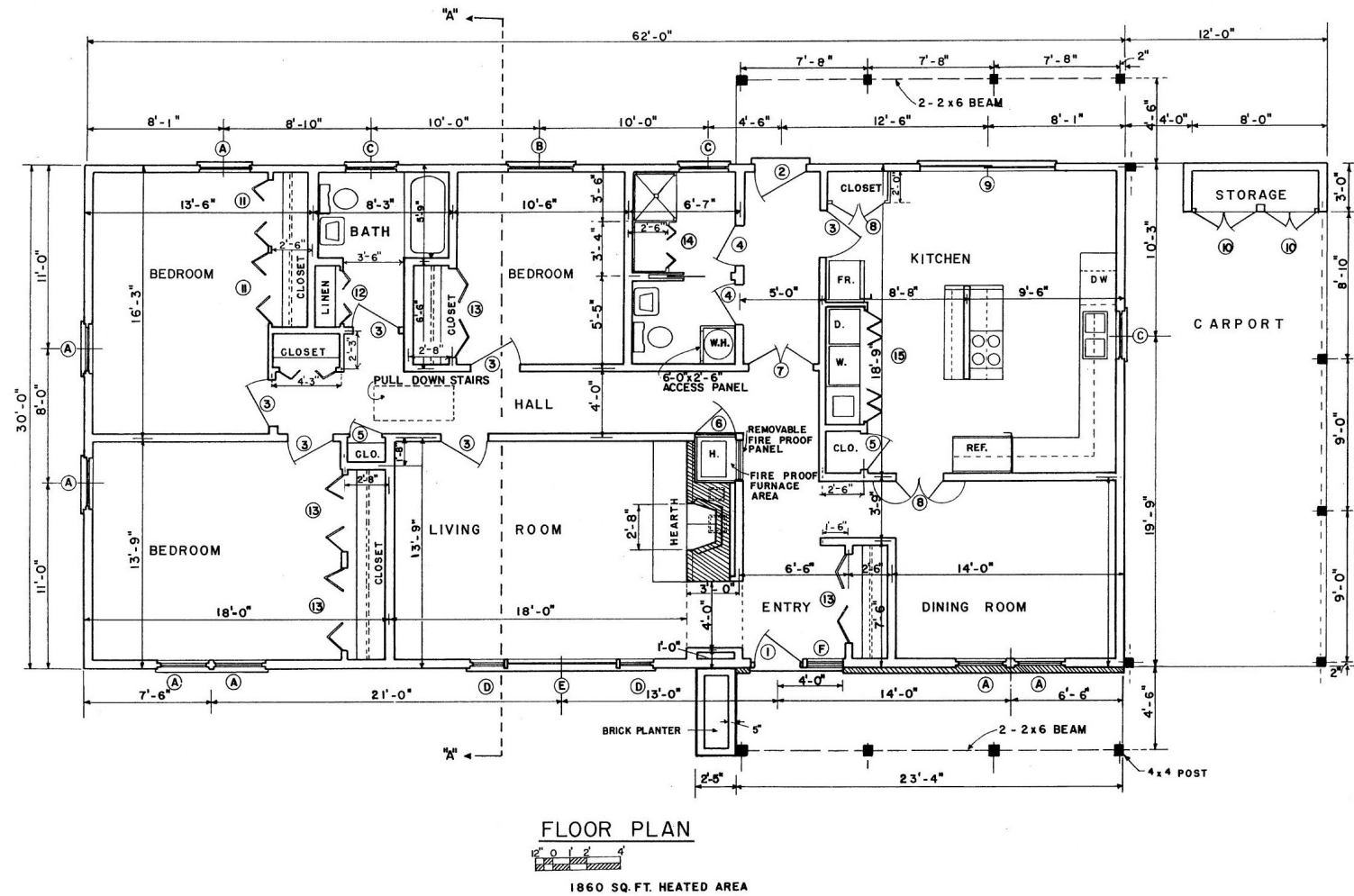
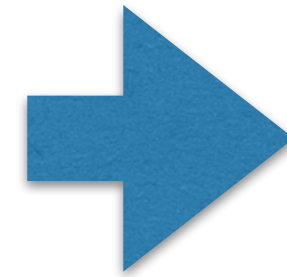
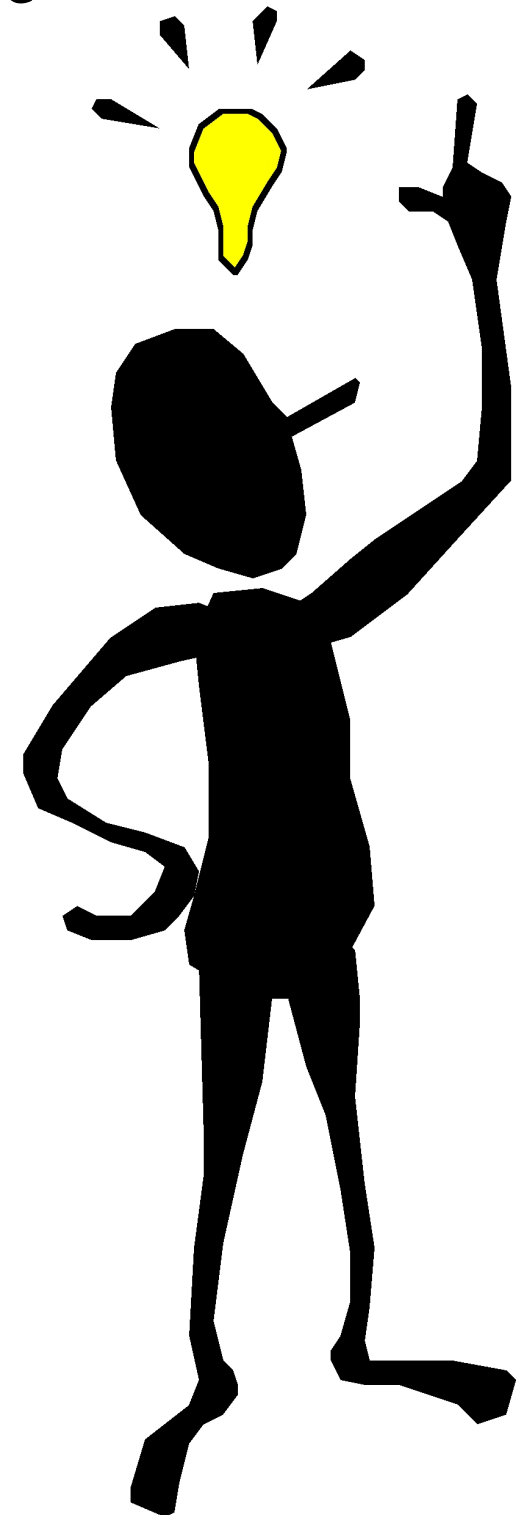
# What is it to design?

the best materials...

low energy

low cost

lots of light



To design is to create a more detailed description of an idea

*Sometimes there is a lot of work between the design and the real thing!*

*Usually many possible solutions!*



**Also simple designs!**

I mix milk, cocoa, and sugar.  
But I am experimenting to  
get the proportions right...



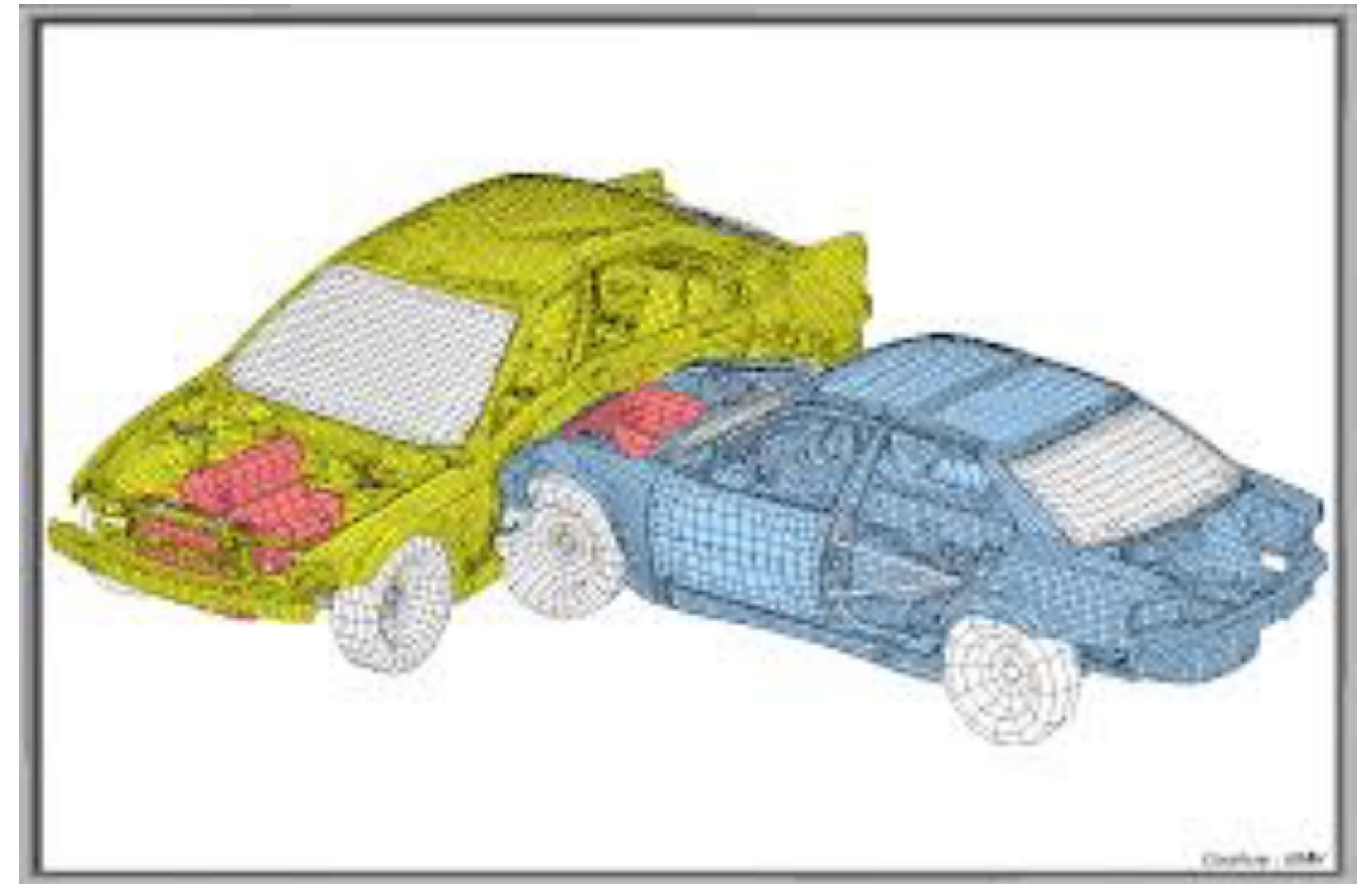
# Common ways to use mathematics in engineering

Energy losses must be reduced through sufficient insulation.



## Dimensioning

(often relatively simple calculations using rules based on experience)

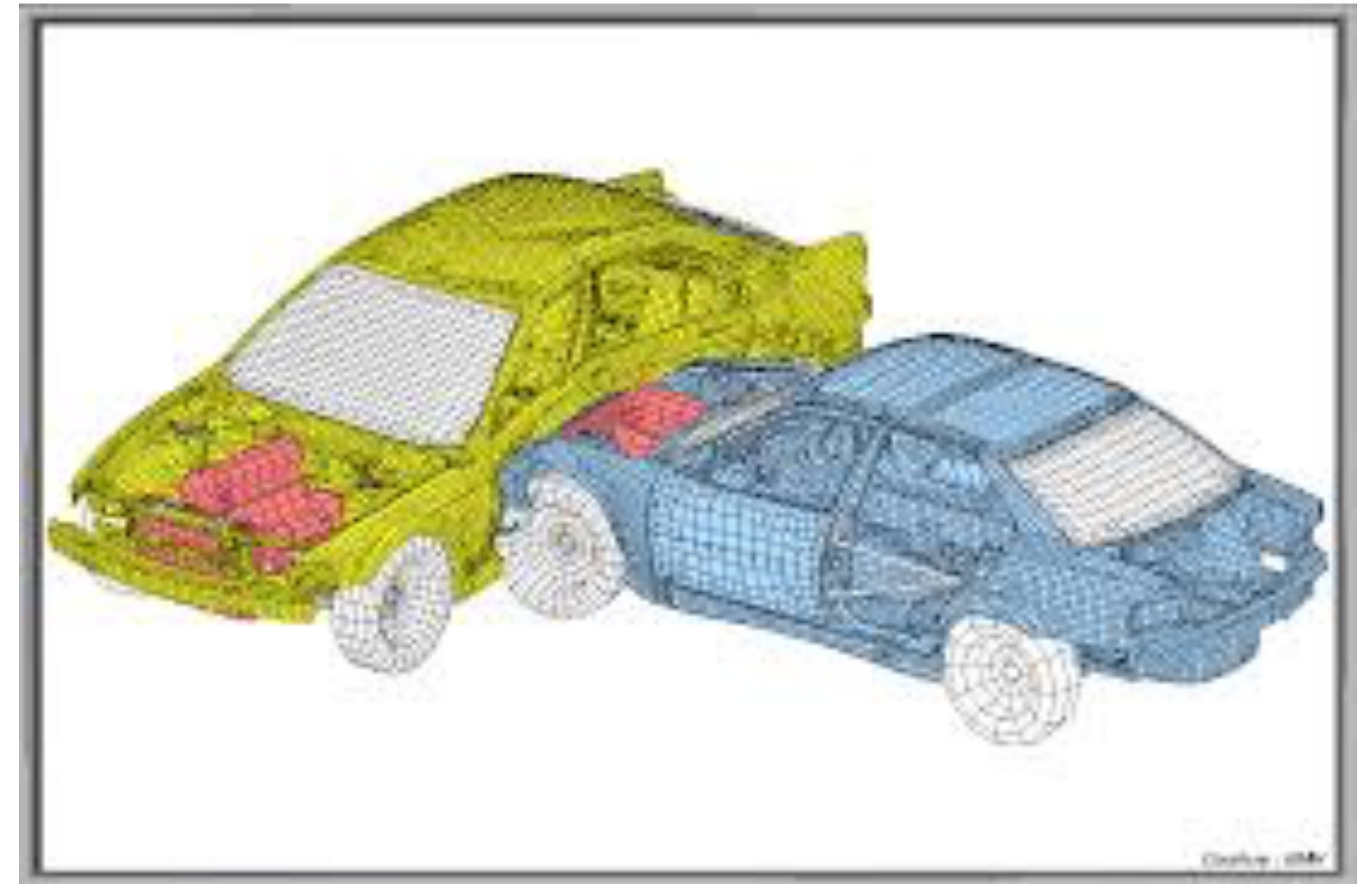
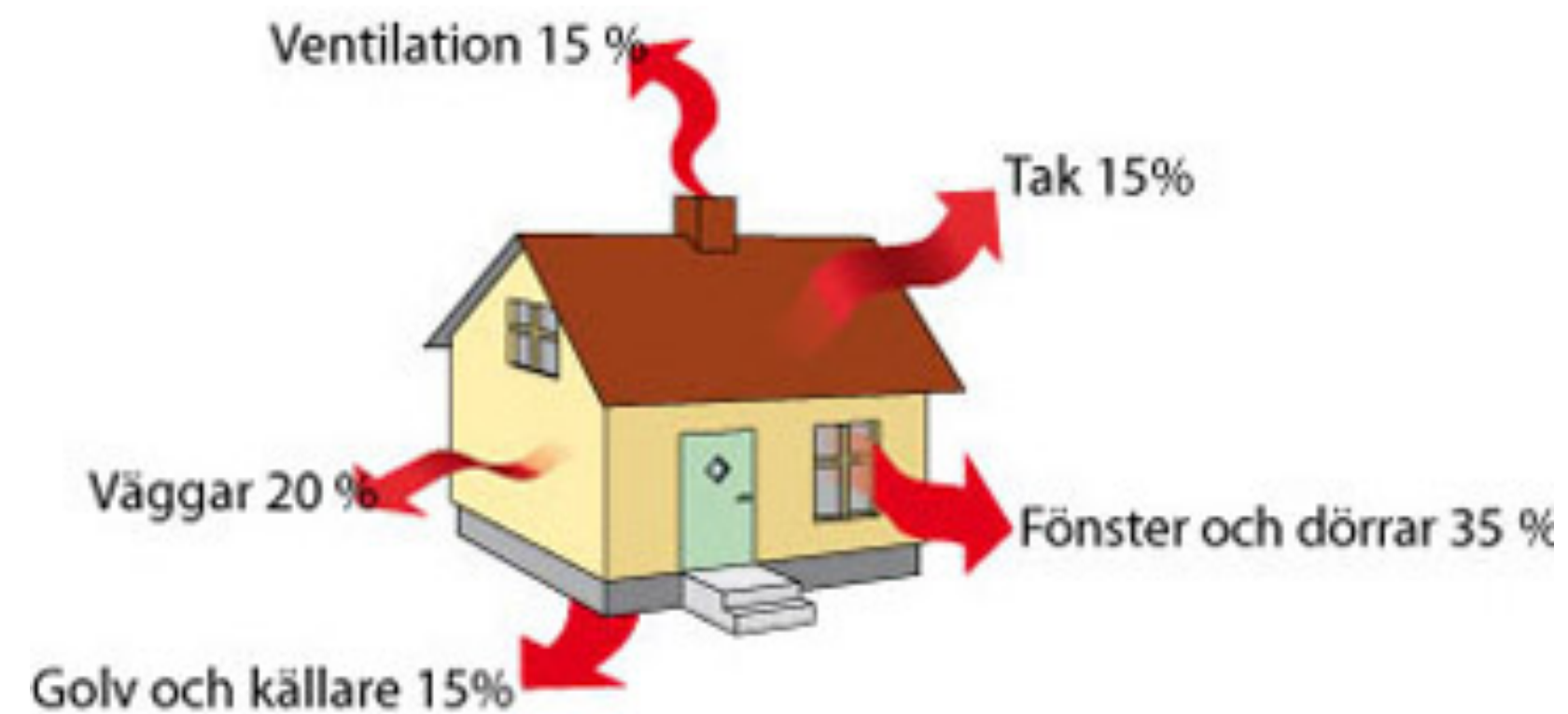


## Simulation

(requires more sophisticated models)

# Common ways to use mathematics in engineering

Energy losses must be reduced through sufficient insulation.



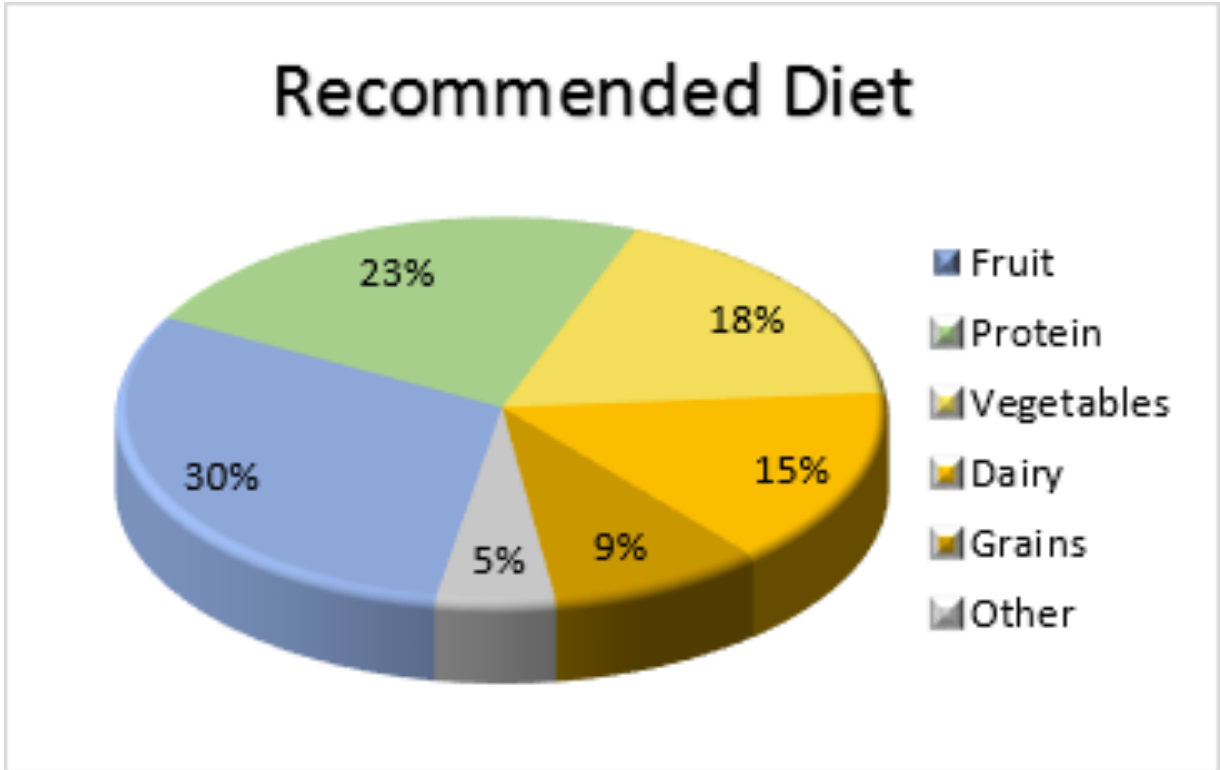
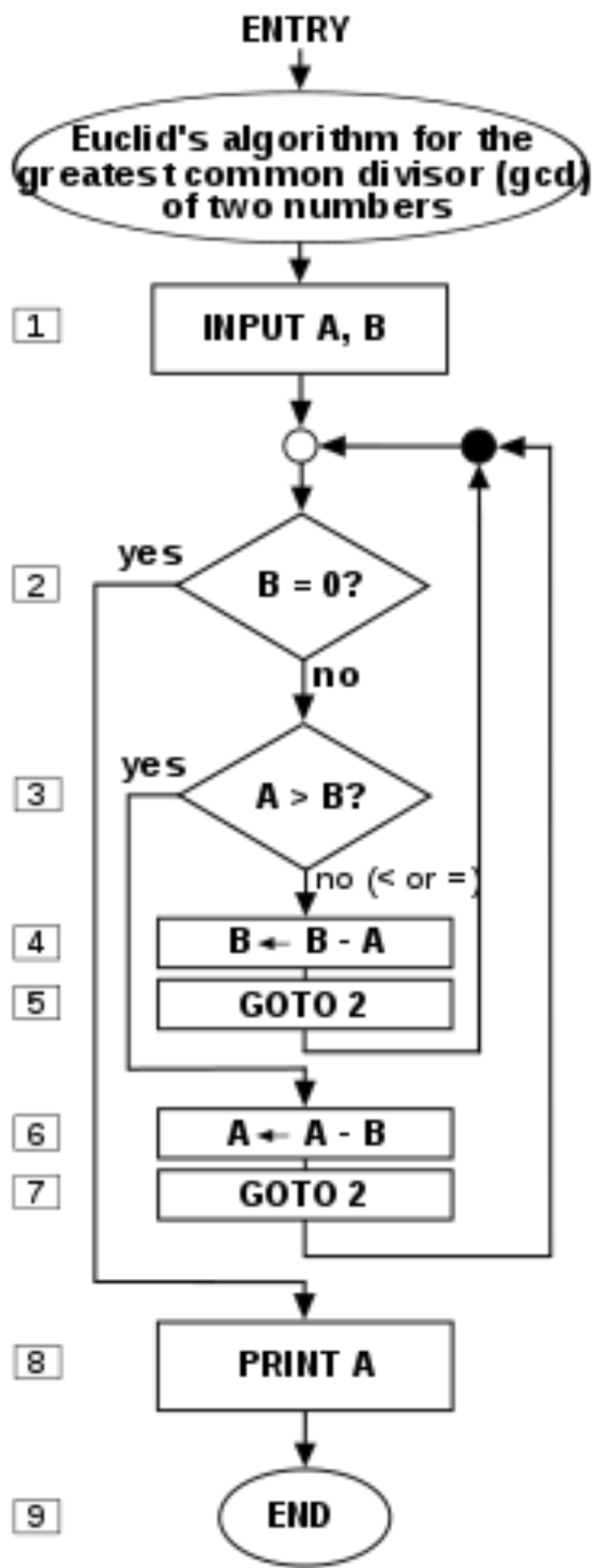
Dimensioning is already a simple form of optimization

Simulation is typically used in a manual process to optimize the design

**You may need measuring to implement the design**



# Abstract mathematical designs



*“the vote needs a 2/3 majority”*

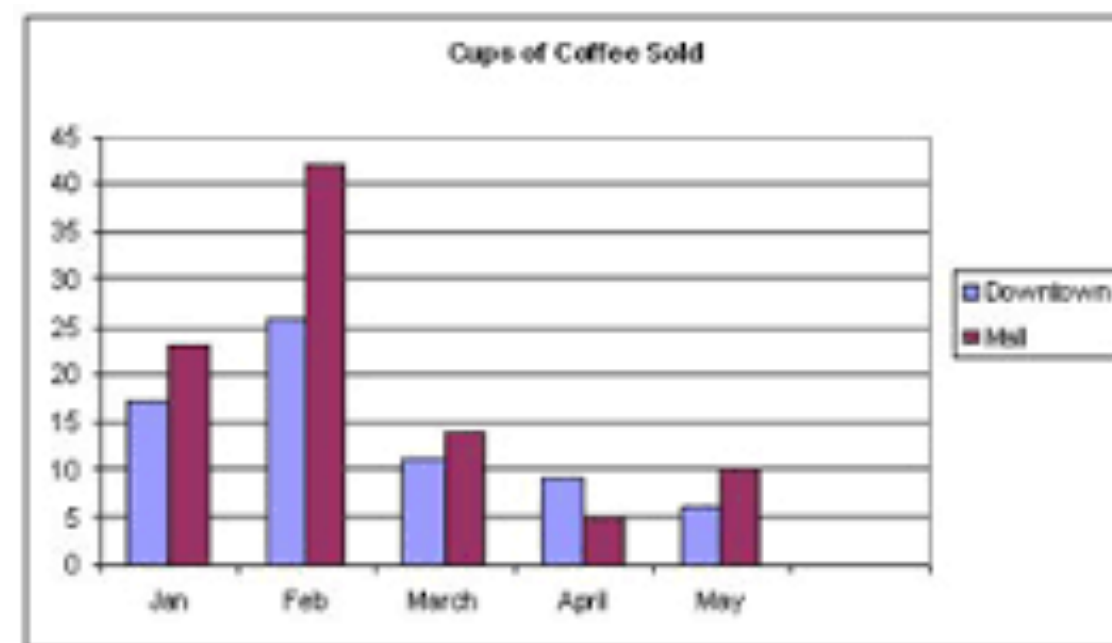
	12PM	1PM	2PM	3PM	4PM	5PM	6PM	7PM	8PM	9PM	10PM		
HARBOURFLAP STAGE	GRUPE ELECTROGENE FANARE CLUB 12:30-13:15	YUNA 14:00-15:00		JOSÉ GONZÁLEZ 15:45-16:45		FAT FREDDY'S DROP 17:15-18:30		LONDON GRAMMAR 19:15-20:15		M.I.A. 21:15-22:30			
FWD STAGE	LOPHIA HANSON ROSKOPICS 13:15-13:30	SENJI LION 13:30-14:15		NO PARTY FOR CHAO DONG 15:00-15:45		...HUNT 16:30-17:30		HYWON 18:15-19:15		BLOOD ORANGE 20:00-21:00	BEI & CHER CHEN 21:45-22:30		
KEF STAGE	TUX 12:45-13:30	FANTASTIC DAY 14:00-14:45		SOI MACHINE 15:15-16:00		GRAND BLANC 16:30-17:15		GOYU 17:45-18:30		PUMAROSA 19:00-19:45	THE TREE & THE WILD 20:15-21:00	66DAYSPROSTATIC 21:30-22:30	
ELECTRIQ STAGE	RUSSELL DOCTROVE (ANGRY CHIEF) 12:00-13:30	JEREMY CHEUNG 13:30-15:00		ASQUARED 15:00-16:30		PUSH 16:30-18:00		LONE 18:00-19:30		OCEAN LAM 19:30-21:00	NICK HÖPNER 21:00-22:30		
ROBOT STAGE		AINESH 13:00-14:30		CRIMES AGAINST POP 14:30-16:00		PODO 16:00-17:30		CLICHE SHOWCASE FT. GONGJIAN & STEPHEN ANDRAKIS 17:30-19:00		MAD PROFESSOR 19:00-20:30	ELIJAH & SKILLIAM 20:30-22:30		
CLUB MINKY		ICHI 13:30-14:15		THE TRAVELING SISTERS 14:45-15:45		YETS DEMON DAVE BAR 16:30-17:30		ALL GENUS ALL IDOT 18:30-19:30		AUNT DONNA 20:00-21:00	THE TUBSAND 21:30-22:30		
ACORN STAGE	ANCESTRAL VOICES 13:00-13:30	RAMBLE & FRIENDS 13:45-14:15		NICK COPE 14:45-15:30		RAMBLE & FRIENDS 15:45-16:30		ICHI 17:00-17:45		TRAVELING SISTERS 18:00-18:30	BLACK BART 19:00-20:00	K-MELO 20:00-21:00	EPIC SHOWCASE FT. MICO, GABRIEL TAN & PRINSEKA 21:00-22:30
SILENT DISCO	YUEN CHI CHUNG 曾智華 AND WAI MAN CHUNG 衛文瑩 12:30-14:30		MANONE ROSE TAYLOR 14:30-15:30		BARNABY BRUCE & CARY ANDERSON 15:30-16:30		JUNIE T. XOXOX 16:30-17:30		TENG BOON EL TAO 17:30-18:30		KAYEMURA TREES 18:30-19:30	SHHWING 19:30-22:00	
CINEMA SILENZIO	SHAIN THE SHEEP 13:00-14:25		THE NEW POLITICS 14:54-15:00		LO AND BEHOLD: SURVIVE OF THE CONNECTED WORLD 15:00-16:38		KIKI 17:00-18:36		SECUL STATION 19:00-20:33				

and many other things...

We will talk more about  
problem solving and the  
design process later in the  
course!

The four categories highlight important aspects of mathematical thinking, and are partially overlapping - but that's ok...

MCMXVII



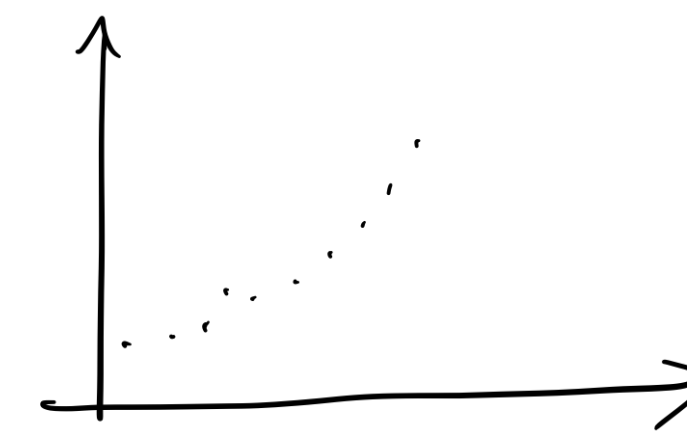
designing for  
keeping track

*definitions*

*proofs*

designing when  
investigating the abstract

*(models)*



$$p = c \cdot e^{at}$$



*better!*

designing when  
investigating the world

*What about theorems?*