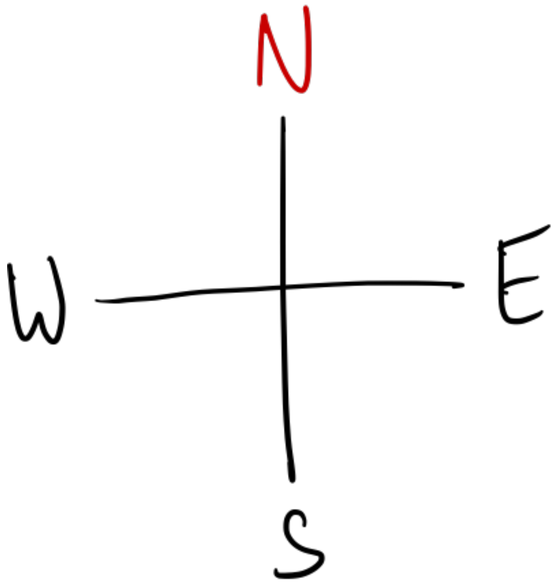
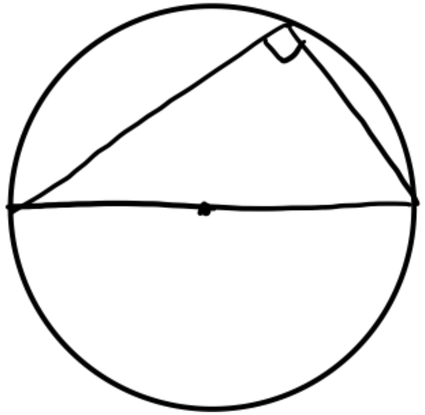
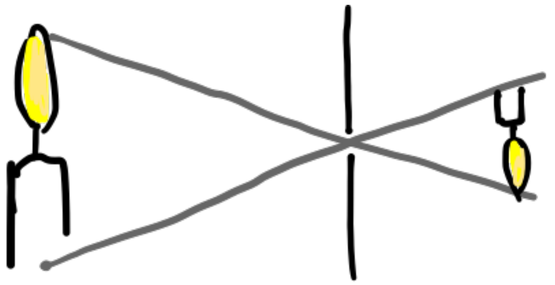
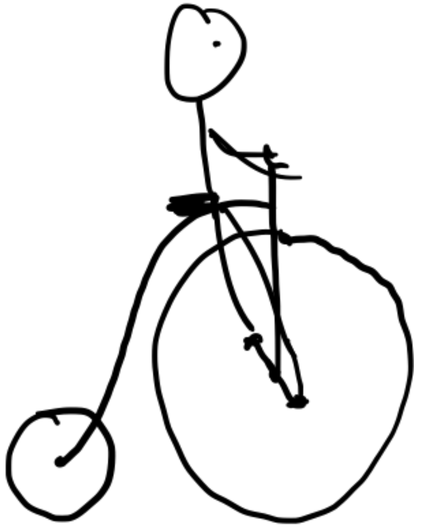
Where and how is mathematical thinking used?



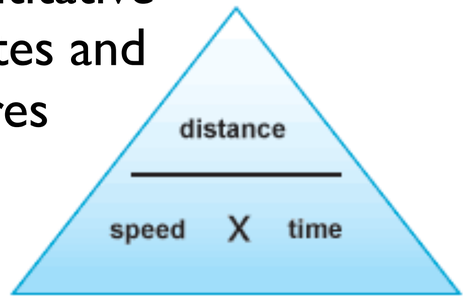






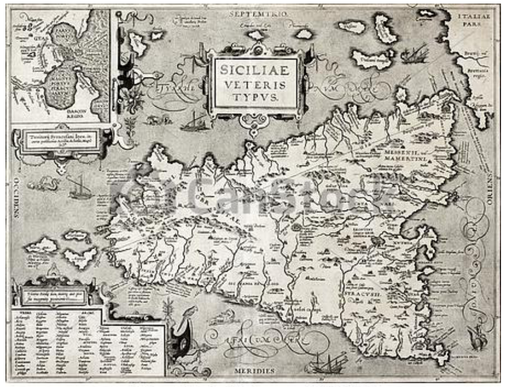
Keeping track

different kinds of quantitative attributes and measures



keeping track is centered around practical needs

"numbers around us"



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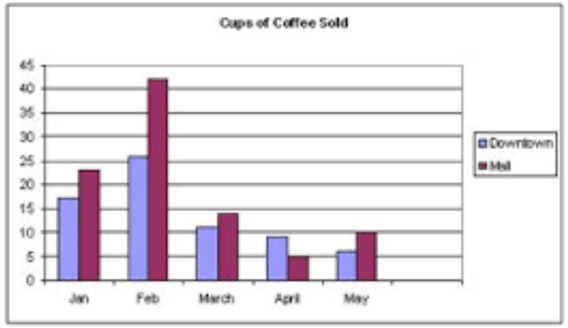


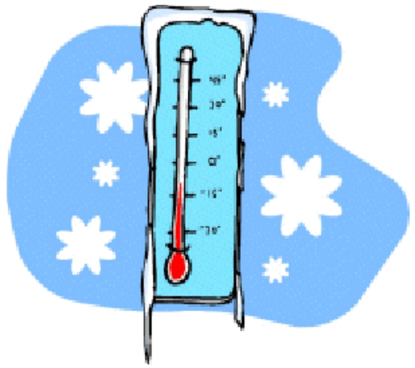
usually nothing interesting is expected - it can be useful but we just keep track

some of this is too basic for this course!

Keeping track of quantities makes it possible to use numbers in many situations…

(qualitative and quantitative understanding)

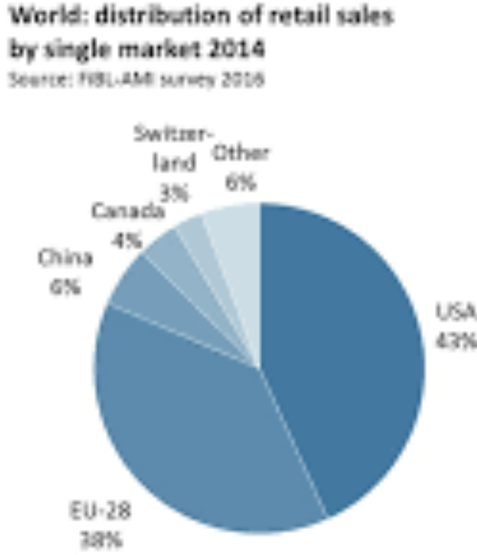
(decisions)



"It's really cold today so I take my warmest coat"

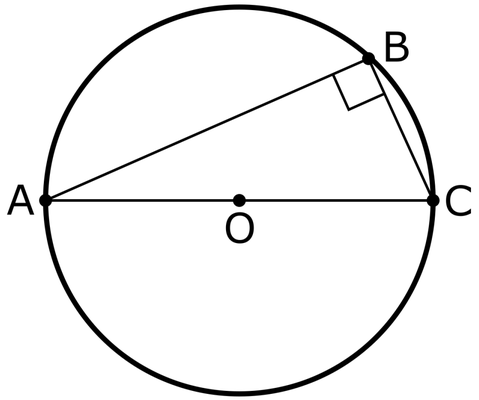
(how much do we need?)

"Fifteen litres of water will be enough"



"Sweden is a small market for organic foods, but large in proportion to its population"

Investigating the abstract



Mathematics itself
(numbers, shapes,…)

Particular abstract problems
(games, strategies, algorithms,…)

Why not just *understanding* the abstract?

To investigate is to carefully examine and draw conclusions.

This is a way to create knowledge.

Investigating is sustainable learning! Just learning other people's solutions is not enough!

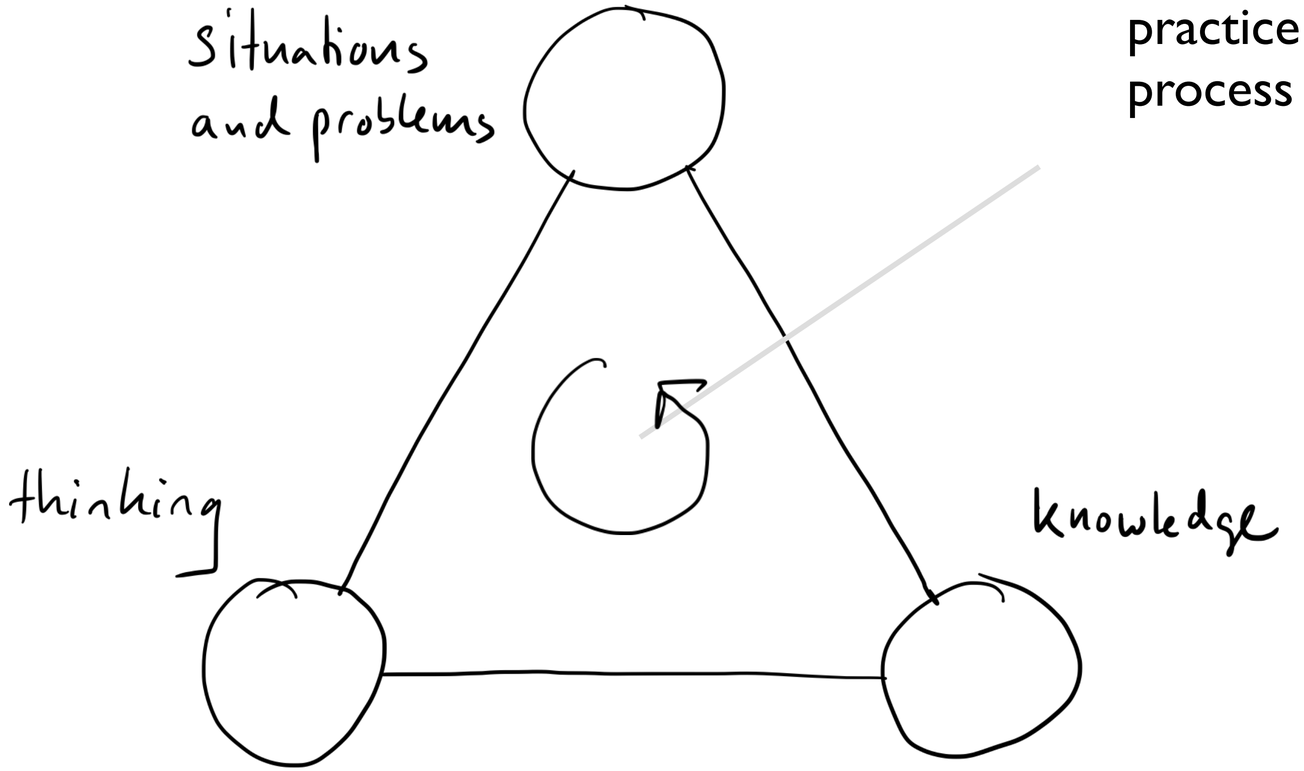
Everyone can learn to investigate, but it requires practice! You practice and extend your natural ability to think

An investigation often begins with a question - and continues with other questions that you formulate along the way

Investigating requires hard work and patience!

"Seek and you shall find!"

consider varied
realistic problems



practice your
thinking

**try to discover
own knowledge**

learn knowledge
found by others

What comes first?

Why do we need to know programming?

So much code has already been written!

What do we mostly do in math education?

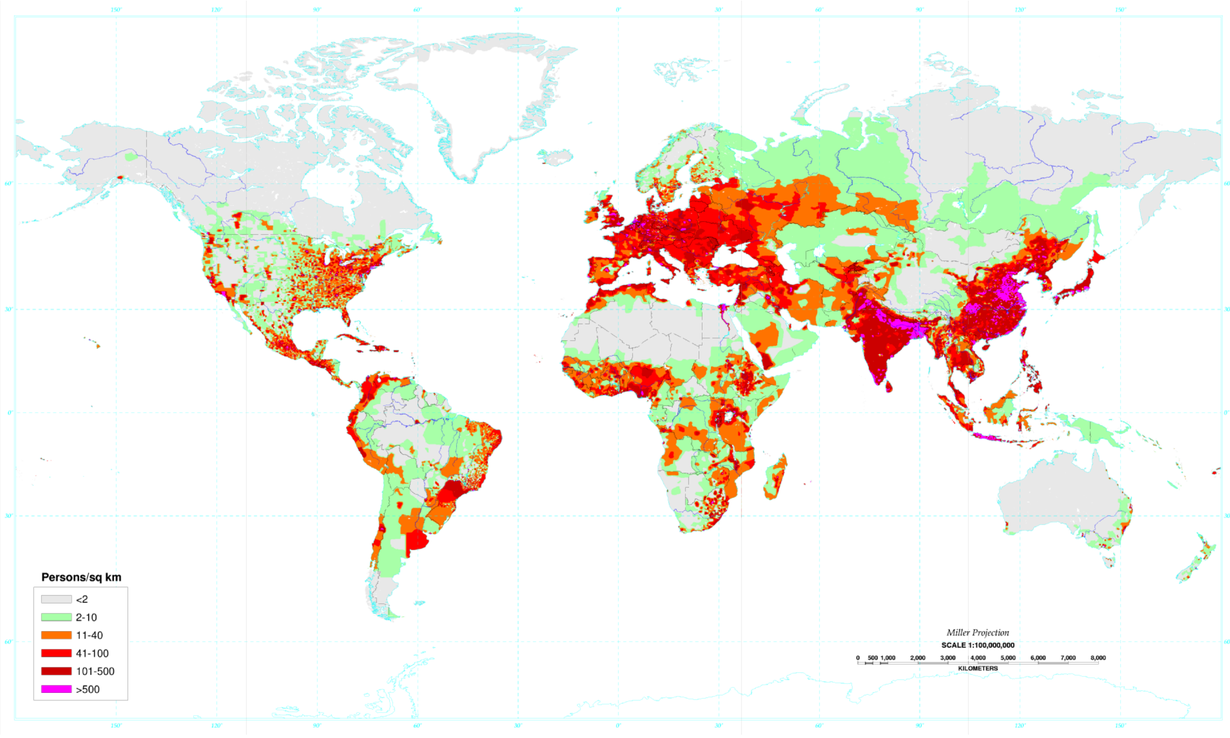
How can I possibly come up with anything new in math with so many smart people working for so long?

"we want students to be able to use the mathematical knowledge they have learned in new situations"
(is this a good way to think?)

investigating the world

Collecting data and empirical modelling

We can learn first hand about the world by collecting data, doing experiments, and drawing conclusions from these



Mechanistic modelling

this is all deductive!

WHY DOES THE MOON FOLLOW US?

If we have some knowledge, we can find out more by just thinking and drawing conclusions

"Primitive" Weather Forecasting Equations

$p = \rho R T$ *Ideal Gas Law* (Equation of State)

$\vec{a}_h = \sum\left(\vec{F}_h / m\right)$ *Newton's Second Law of Motion*

$\Delta p = -\rho g \Delta z$

$(PGA)_v = g$

$\vec{a}_v = \sum(\vec{F}_v / m) = (\vec{P}\vec{G}\vec{A})_v - \vec{g}$

Hydrostatic Law (Obtained from the Equation of Vertical Motion)

$\Delta T = \Delta q / c_p + (1/\rho)\Delta p$ *First Law of Thermodynamics*

$(1/\rho)\Delta\rho/\Delta t = -DIV$

Conservation of Mass Applied to the Atmosphere (Equation of Continuity)

Zonal wind:

$$\frac{\partial u}{\partial t} = \eta v - \frac{\partial \Phi}{\partial x} - c_p\theta\frac{\partial \pi}{\partial x} - z\frac{\partial u}{\partial \sigma} - \frac{\partial(\frac{u^2+v^2}{2})}{\partial x}$$

Meridional wind:

$$\frac{\partial v}{\partial t} = -\eta\frac{u}{v} - \frac{\partial \Phi}{\partial y} - c_p\theta\frac{\partial \pi}{\partial y} - z\frac{\partial v}{\partial \sigma} - \frac{\partial(\frac{u^2+v^2}{2})}{\partial y}$$

Temperature:

$$\frac{\delta T}{\delta t} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

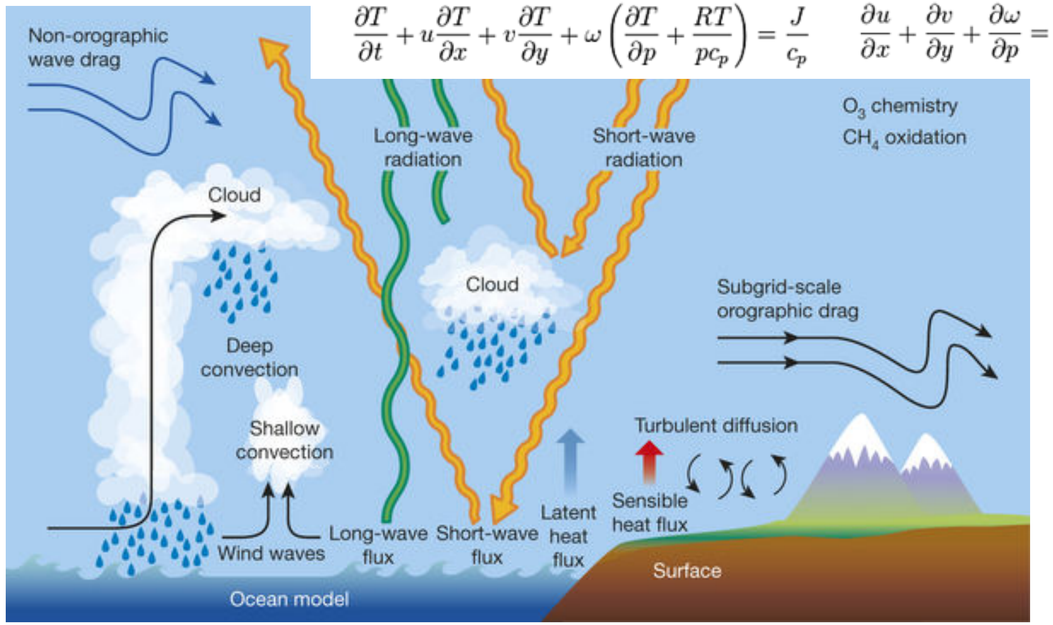
Precipitable water:

$$\frac{\delta W}{\delta t} = u\frac{\partial W}{\partial x} + v\frac{\partial W}{\partial y} + w\frac{\partial W}{\partial z}$$

Pressure thickness:

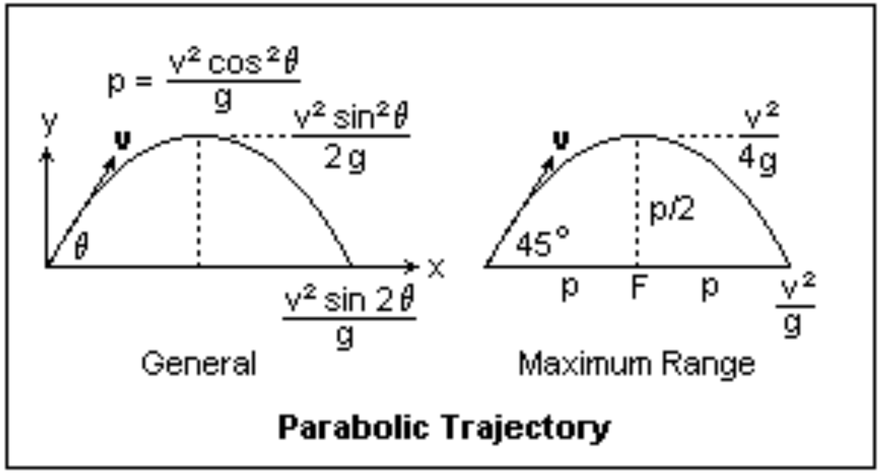
$$\frac{\partial}{\partial t}\frac{\partial p}{\partial \sigma} = u\frac{\partial}{\partial x}x\frac{\partial p}{\partial \sigma} + v\frac{\partial}{\partial y}y\frac{\partial p}{\partial \sigma} + w\frac{\partial}{\partial z}z\frac{\partial p}{\partial \sigma}$$

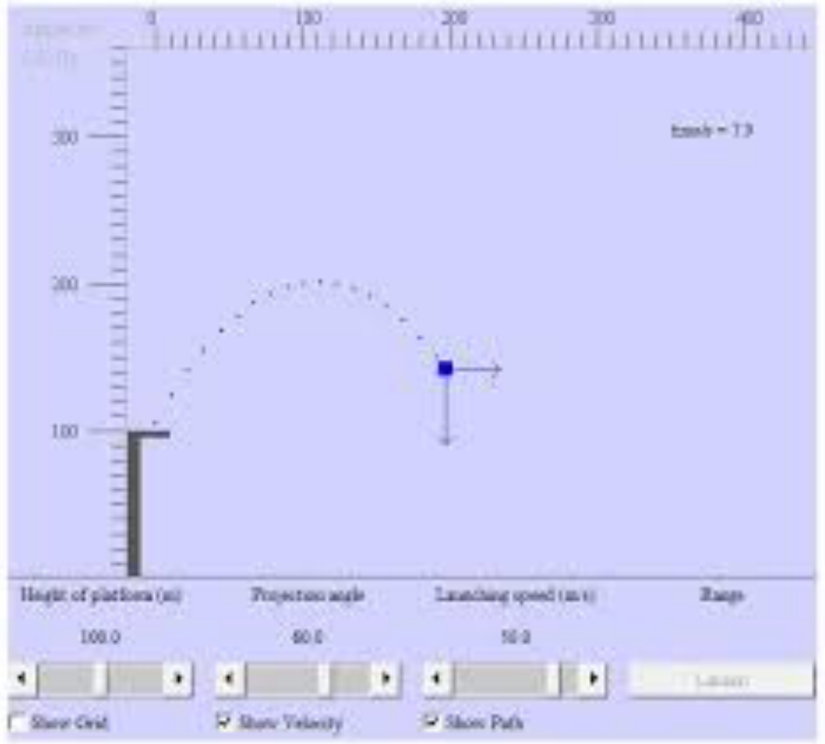
$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \omega\left(\frac{\partial T}{\partial p} + \frac{RT}{pc_p}\right) = \frac{J}{c_p} \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \qquad 0 = -\frac{\partial \phi}{\partial p} - \frac{RT}{p}$$



If we know well how things work we can create models that simulate reality in the computer and learn even more!

Two fundamentally different ways to do calculations about the real world





Simple direct calculation for easy things

Step by step simulation is feasible for much more complicated systems!

Qualitative understanding is more important than full quantitative understanding

"there are atoms and molecules"

"the apple falls to the ground"

"this culture of borrowing is not healthy"

"basically, the population increases in proportion to its size"

In research, quantitative investigation can lead to significant qualitative understanding

When you do calculations, never lose track of your understanding of the corresponding reality!

Many aspects of nature can be explained surprisingly well with mathematics!

Why?

What can, and what cannot, be explained well with math?

designing

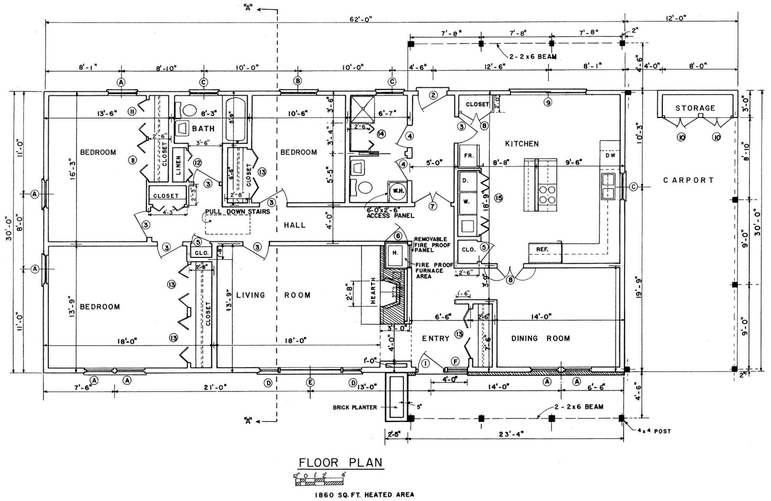
What is it to design?

the best materials…

low energy

low cost

lots of light



To design is to create a more detailed description of an idea

Usually <u>many</u> possible solutions!

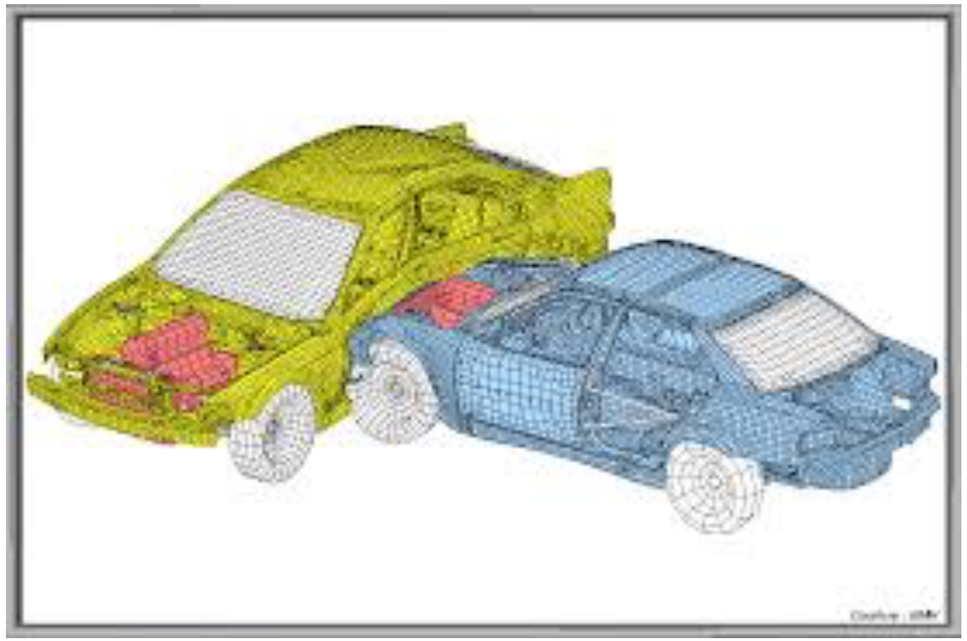
Sometimes there is a lot of work between the design and the real thing!

Also simple designs!

I mix milk, cocoa, and sugar.
But I am experimenting to
get the proportions right…

Common ways to use mathematics in engineering

Energy losses must be reduced through sufficient insulation.

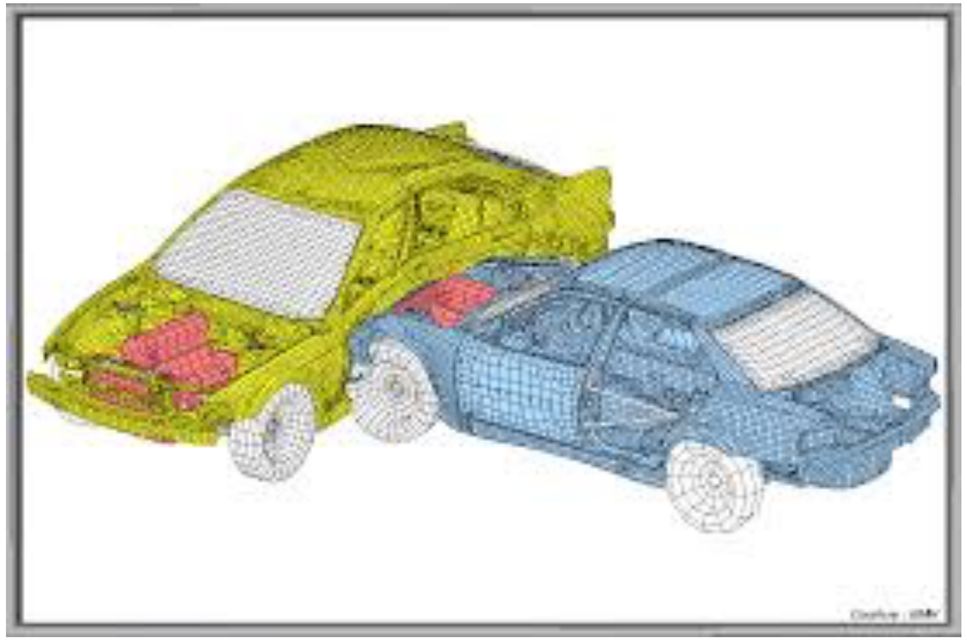


Dimensioning
(often relatively simple calculations using rules based on experience)

Simulation
(requires more sophisticated models)

Common ways to use mathematics in engineering

Energy losses must be reduced through sufficient insulation.

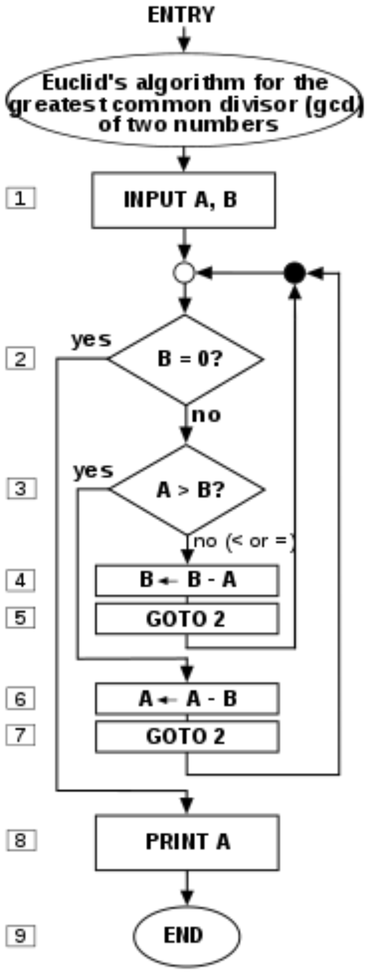


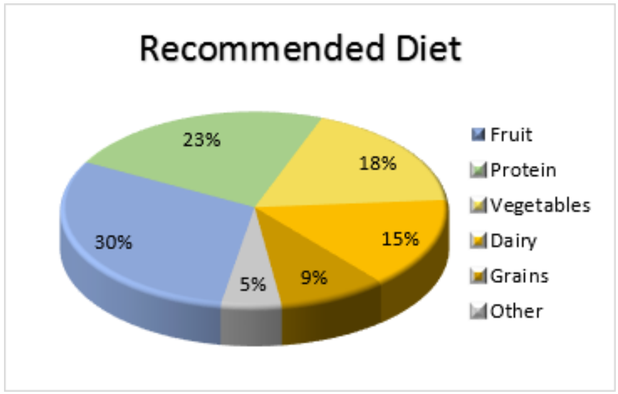
Dimensioning is already a simple form of optimization

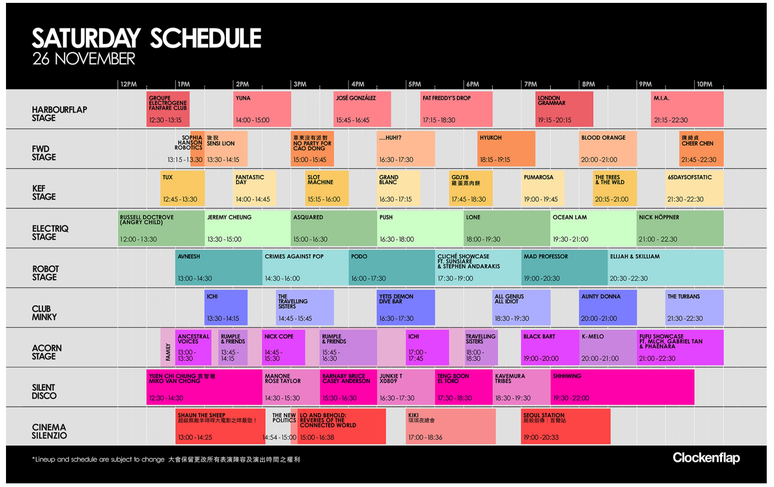
Simulation is typically used in a manual process to optimize the design

You may need measuring to implement the design

Abstract mathematical designs







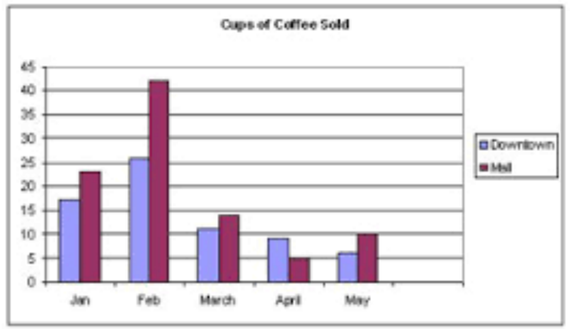
"the vote needs a 2/3 majority"

and many other things…

We will talk more about problem solving and the design process later in the course!

The four categories highlight important aspects of mathematical thinking, and are partially overlapping - but that's ok…

MCMXVII



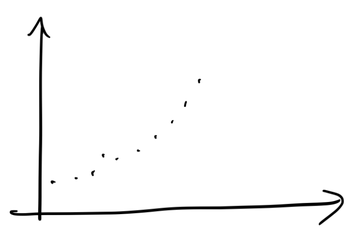
designing for
keeping track

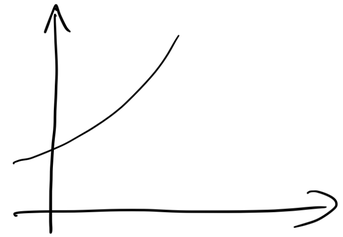
definitions

proofs

designing when
investigating the abstract

(models)





$p = c \cdot e^{at}$

better?

designing when
investigating the world

What about theorems?