Mathematical thinking: a glance at the history

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Dag Wedelin 2018 (updates)

Origins

How did it start?

- innate sense of numbers & distances
- counting & recording
- making tools and building

What drove it?

- survival
- curiosity
- control

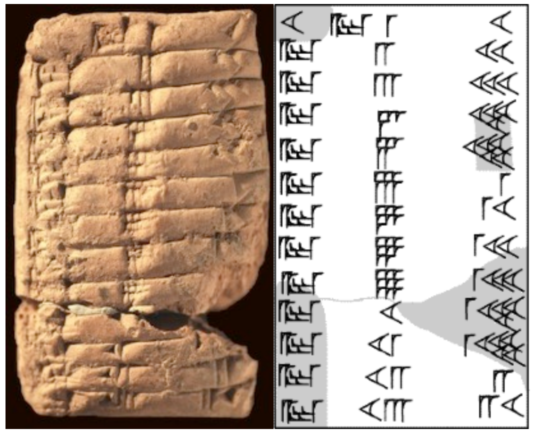
Number notations

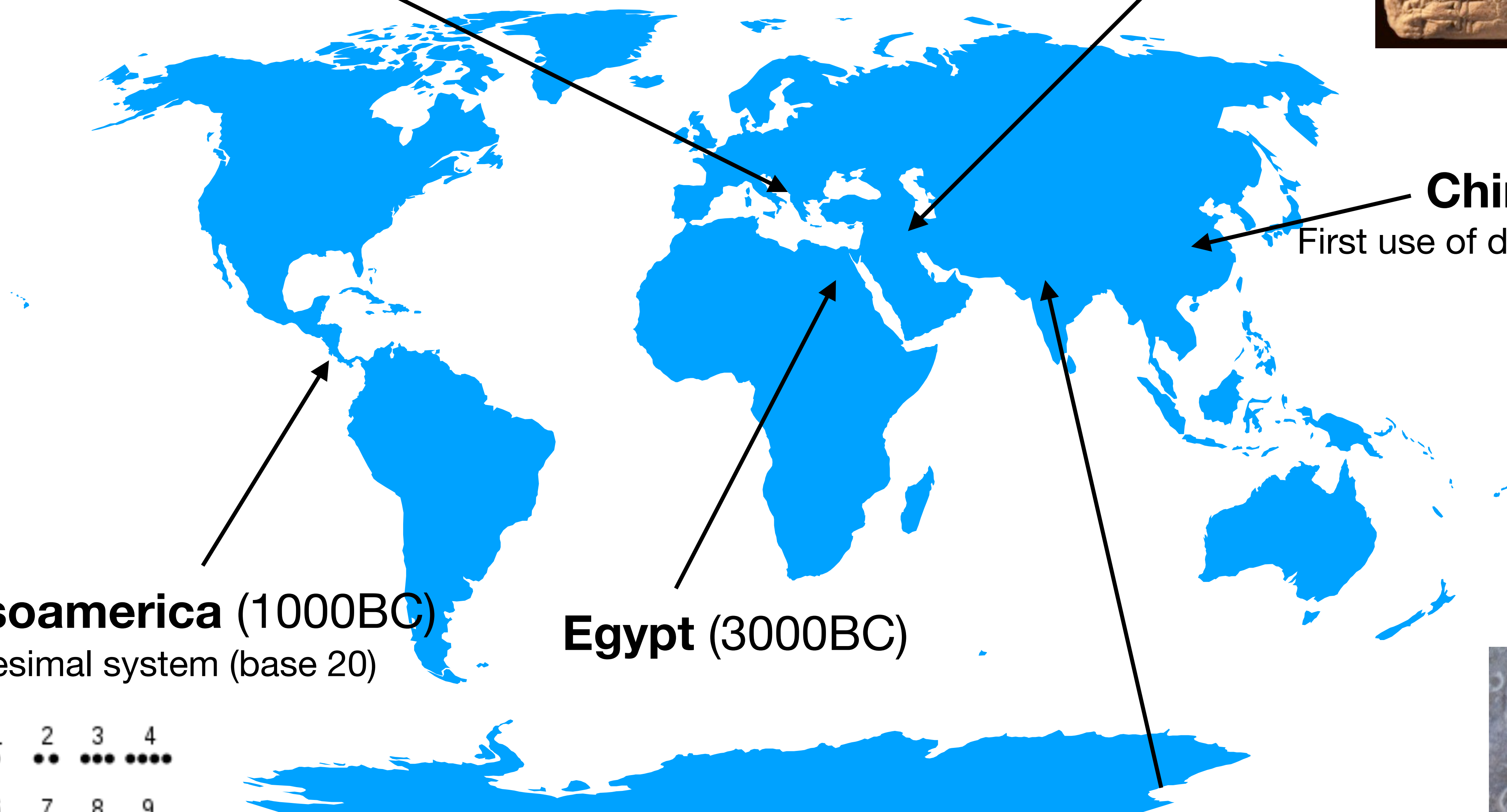
Greco-Roman (1st mil BC)

- I, II, III, IIII, Γ, ΓI, ΓII, ΓIII, ΓIIII, Δ
- I, II, III, IV, V, VI ,VII, VIII, IX, X

Mesopotamia (2000 BC)

- two symbols: 𒁹 = 1 𒌋 = 10
- base 60
- place value 𒁹 𒁹 𒁹 = 60×60+60+1



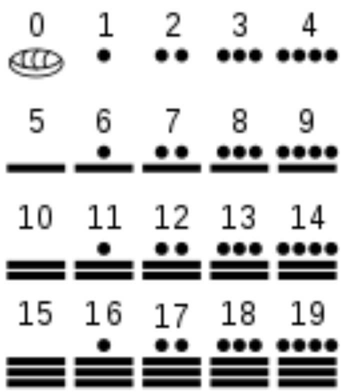


China (ca 475BC)

First use of decimal place-value system

Mesoamerica (1000BC)

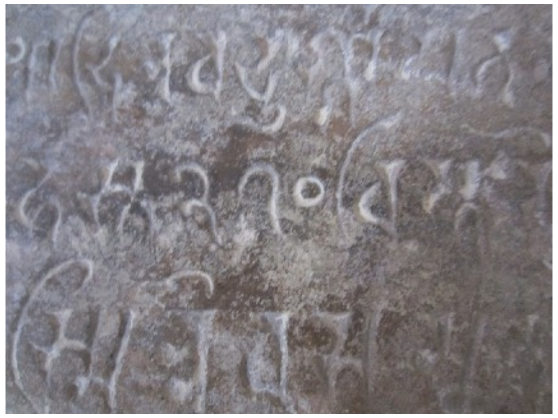
- vigesimal system (base 20)



Egypt (3000BC)

India (1C AD)

- First appearance of zero symbol
- Basis for our number system



Arithmetic — Egypt

Hieroglyphic numerals

1

10

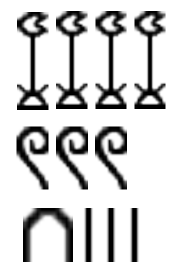
100

1000

10000

100000

1000000



= 4313

Multiplication

8×7=8×(1+2+2×2)

Division

'Bread and Beer problem'

If you are told: a truncated pyramid of 6 for the vertical height by 4 on the base by 2 on the top: You are to square the 4; result 16. You are to double 4; result 8. You are to square this 2; result 4. You are to add the 16 and the 8 and the 4; result 28. You are to take 1/3 of 6; result 2. You are to take 28 twice; result 56. See, it is of 56. You will find [it] right.

— Moscow papyrus, ca. 1850 BC

— Rhind papyrus, 1550 BC

Arithmetic — Babylonia

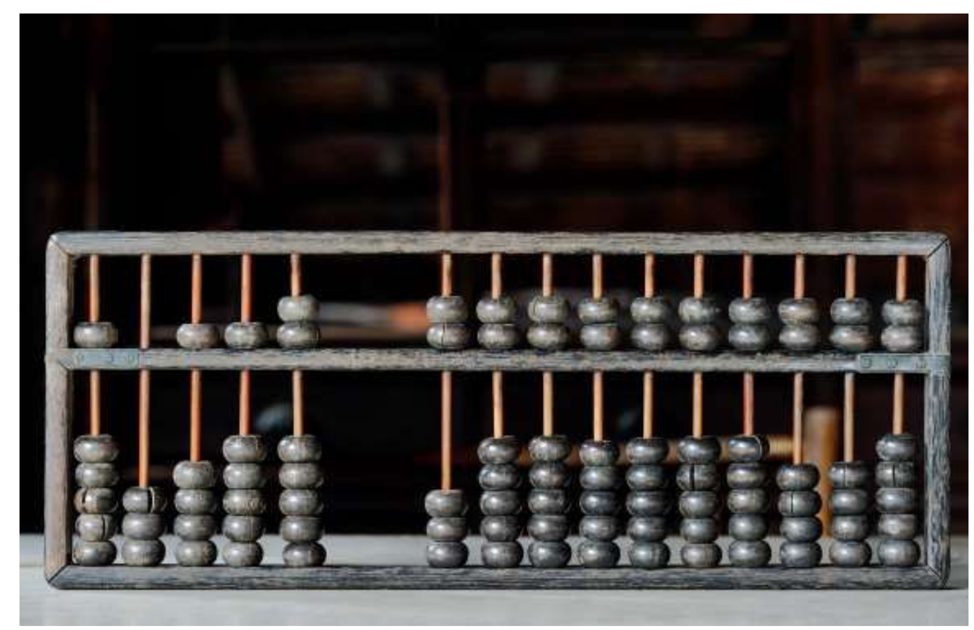
Ca 1800 BC

Multiplication with quadratic tables

$$ab = \frac{(a+b)^2 - (a-b)^2}{4}$$

Division with inversion tables

$$\frac{a}{b} = a \cdot \frac{1}{b}$$



Zero as a placeholder

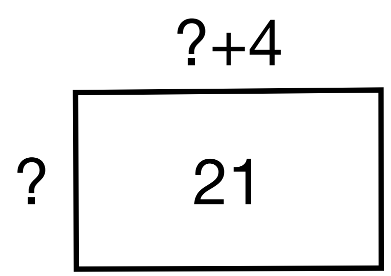
Finding unknowns (reverse arithmetic)

Linear equation

Egyptian Aha problems:

a quantity taken 1 and ½ times
and added to 4 to make 10

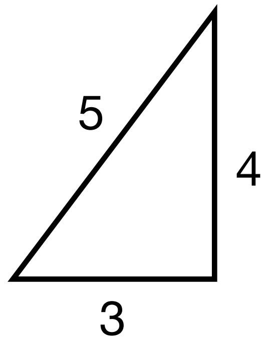
Babylonian irrigation and land management



Babylonian version of Pythagoras theorem:
4 is the length and 5 the diagonal. What is the breadth?
Its size is not known.
4 times 4 is 16.
5 times 5 is 25.
You take 16 from 25 and there remains 9.
What times what shall I take in order to get 9?
3 times 3 is 9.
3 is the breadth.

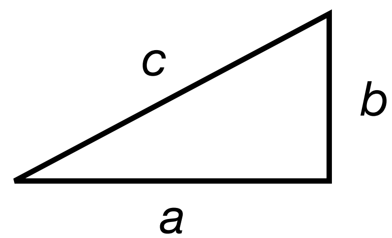
Quadratic equation

The power of proof (Greece)



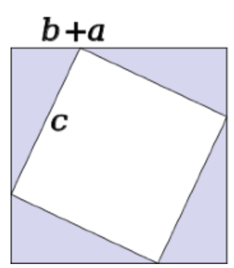
Plimpton 322 tablet: 15 perfect right-angled triangles (Babylonian ca 1800BC)

Pythagoras' Theorem:
For <u>every</u> right-angled triangle

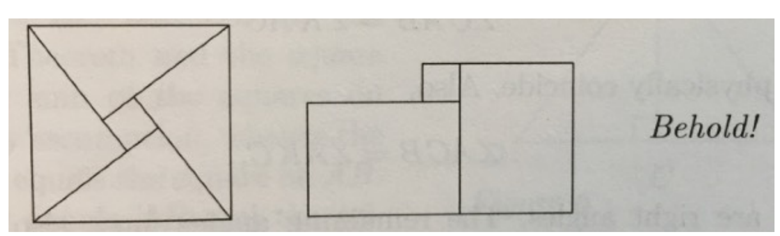


we have $a^2 + b^2 = c^2$

An algebraic proof



Re-arrangement



The most important contribution of ancient Greek mathematics is systematic proof

Euclid's Elements
about 300BC

mostly geometry and numbers

New modern structure: definition, axiom, theorem, proof

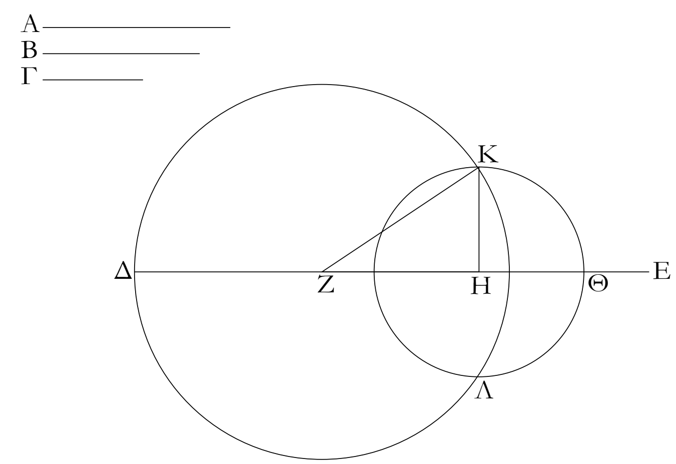
One of the most famous books of all time

ΓΕΒ μείζων ἐστὶ τῆς ὑπὸ ΒΑΓ. ἀλλὰ τῆς ὑπὸ ΓΕΒ μείζων ἐδείχθη ἡ ὑπὸ ΒΔΓ· πολλῷ ἄρα ἡ ὑπὸ ΒΔΓ μείζων ἐστὶ τῆς ὑπὸ ΒΑΓ.

Ἐὰν ἄρα τριγώνου ἐπὶ μιᾶς τῶν πλευρῶν ἀπὸ τῶν περάτων δύο εὐθεῖαι ἐντὸς συσταθῶσιν, αἱ συσταθεῖσαι τῶν λοιπῶν τοῦ τριγώνου δύο πλευρῶν ἐλάττονες μέν εἰσιν, μείζονα δὲ γωνίαν περιέχουσιν· ὅπερ ἔδει δεῖξαι.

κβ΄.

Ἐκ τριῶν εὐθειῶν, αἵ εἰσιν ἴσαι τρισὶ ταῖς δοθείσαις [εὐθείαις], τρίγωνον συστήσασθαι· δεῖ δὲ τὰς δύο τῆς λοιπῆς μείζονας εἶναι πάντῃ μεταλαμβανομένας [διὰ τὸ καὶ παντὸς τριγώνου τὰς δύο πλευρὰς τῆς λοιπῆς μείζονας εἶναι πάντῃ μεταλαμβανομένας].



Ἔστωσαν αἱ δοθεῖσαι τρεῖς εὐθεῖαι αἱ Α, Β, Γ, ὧν αἱ δύο τῆς λοιπῆς μείζονες ἔστωσαν πάντῃ μεταλαμβανόμεναι, αἱ μὲν Α, Β τῆς Γ, αἱ δὲ Α, Γ τῆς Β, καὶ ἔτι αἱ Β, Γ τῆς Α· δεῖ δὴ ἐκ τῶν ἴσων ταῖς Α, Β, Γ τρίγωνον συστήσασθαι.

Ἐκκείσθω τις εὐθεῖα ἡ ΔΕ πεπερασμένη μὲν κατὰ τὸ Δ ἄπειρος δὲ κατὰ τὸ Ε, καὶ κείσθω τῇ μὲν Α ἴση ἡ ΔΖ, τῇ δὲ Β ἴση ἡ ΖΗ, τῇ δὲ Γ ἴση ἡ ΗΘ· καὶ κέντρῳ μὲν τῷ Ζ, διαστήματι δὲ τῷ ΖΔ κύκλος γεγράφθω ὁ ΔΚΛ· πάλιν κέντρῳ μὲν τῷ Η, διαστήματι δὲ τῷ ΗΘ κύκλος γεγράφθω ὁ ΚΛΘ, καὶ ἐπεζεύχθωσαν αἱ ΚΖ, ΚΗ· λέγω, ὅτι ἐκ τριῶν εὐθειῶν τῶν ἴσων ταῖς Α, Β, Γ τρίγωνον συνέσταται τὸ ΚΖΗ.

Ἐπεὶ γὰρ τὸ Ζ σημεῖον κέντρον ἐστὶ τοῦ ΔΚΛ κύκλου, ἴση ἐστὶν ἡ ΖΔ τῇ ΖΚ· ἀλλὰ ἡ ΖΔ τῇ Α ἐστιν ἴση. καὶ ἡ

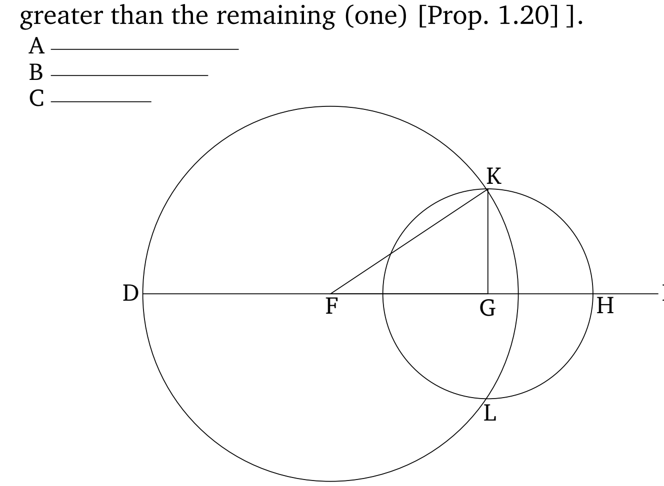
(the sum of) BD and DC.

Again, since in any triangle the external angle is greater than the internal and opposite (angles) [Prop. 1.16], in triangle CDE the external angle BDC is thus greater than CED. Accordingly, for the same (reason), the external angle CEB of the triangle ABE is also greater than BAC. But, BDC was shown (to be) greater than CEB. Thus, BDC is much greater than BAC.

Thus, if two internal straight-lines are constructed on one of the sides of a triangle, from its ends, the constructed (straight-lines) are less than the two remaining sides of the triangle, but encompass a greater angle. (Which is) the very thing it was required to show.

Proposition 22

To construct a triangle from three straight-lines which are equal to three given [straight-lines]. It is necessary for (the sum of) two (of the straight-lines) taken together in any (possible way) to be greater than the remaining (one), [on account of the (fact that) in any triangle (the sum of) two sides taken together in any (possible way) is greater than the remaining (one) [Prop. 1.20]].



Let A, B, and C be the three given straight-lines, of which let (the sum of) two taken together in any (possible way) be greater than the remaining (one). (Thus), (the sum of) A and B (is greater) than C, (the sum of) A and C than B, and also (the sum of) B and C than A. So it is required to construct a triangle from (straight-lines) equal to A, B, and C.

Let some straight-line DE be set out, terminated at D, and infinite in the direction of E. And let DF made equal to A, and FG equal to B, and GH equal to C [Prop. 1.3]. And let the circle DKL have been drawn with center F and radius FD. Again, let the circle KLH have been drawn with center G and radius GH. And let KF and KG have been joined. I say that the triangle KFG has

Medieval mathematics

New numbers from India: (7th C AD)

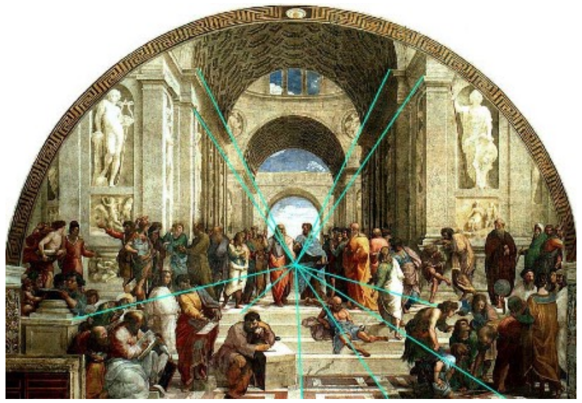
- Properties of 0. 1+0 = 1, 1-0 = 1, 1×0= 0
- 1 ÷ 0 = ?
- Negative numbers, a new sort of nothing
- Playing with infinity

$$\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \cdots$$

Algebra from Middle East (9th C AD):

"$x^2 = 40x - 4x^2$" is transformed by ***al-ğabr*** into "$5x^2 = 40x$"
completion or "reunion of broken parts"
(but described in ordinary sentences)

New maths *13th—18th C*



Art of Perspective

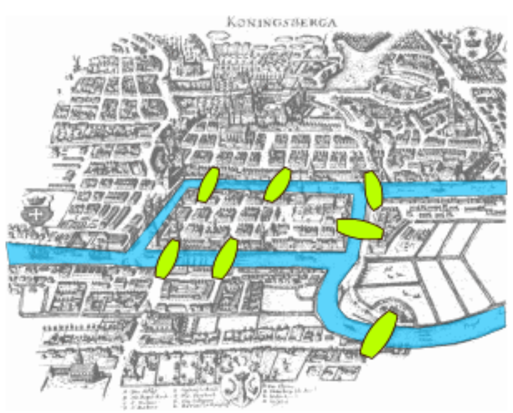
$$\frac{3x}{2}+4=10$$

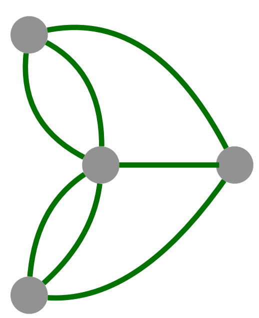
Algebraic notation

$$x(x+4)=21$$

Logarithms

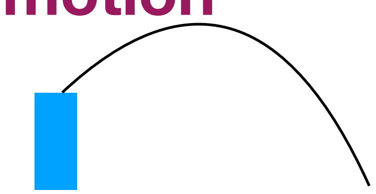
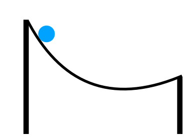
Structure and Form





Bridges of Königsberg

Calculus of motion

- Trajectories 
- Bernoulli problem 
- Fundamental Theorem of Calculus: $f=\frac{d}{dx}\int f\,dx$

Changing perspective *16th—19th C*

New numbers

$$i = \sqrt{-1}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{1}{1} + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3 \times 2} + \cdots$$

A marriage of concepts

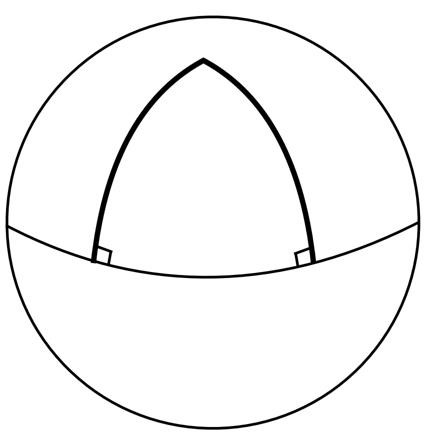
- Algebra & geometry

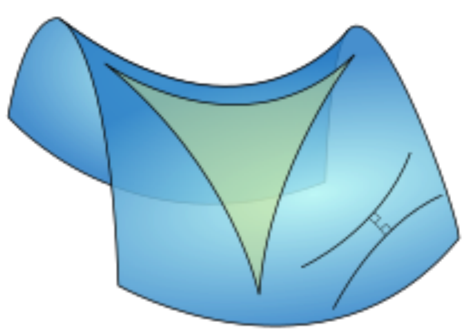
$y = ax^2$

$x^2 + y^2 = r^2$

- A unity of numbers $e^{i\pi} = -1$

Non-Euclidean geometry





New ways of counting

A broadening horizon

Statistics & probability (from 17th C)

Computer science (20th C)

optimization

numerical analysis

many applied areas

Mathematics is our oldest science!