7 Discrete models

Depending on your background, some of these exercises may be easy, while others may be more difficult. We simply ask you to do your best with these problems, within the limits of the time allocated for the course.

(keeping track)

(USING BASIC DISCRETE STRUCTURES)

Give applied examples of how you can represent or organize data (or procedures), in terms of each of the following basic mathematical concepts:

Set: unordered collection of items.

Sequence: sequentially ordered collection of items.

Tree: branching structure with a root.

Graph: Structure with vertices and edges

Directed graph: graph where links are directed (usually represented with arrows)

Weighted graph: graph with a single number for each edge (can be undirected or directed)

(MATHEMATICS VERSUS DATA STRUCTURES)

What are your thoughts about the relationship between discrete mathematical concepts as these mentioned above, and data structures in programming?

(investigating the abstract)

(MAP COLOURING)

This problem is about how many colours you may need to colour a map so that neighbouring countries have different colours.

a) Suggest how to model this problem as a graph problem (i.e. as a problem on a graph).

b) Try to figure out how many colours you may need to colour any map (a finite number or possibly infinitely many)? Create examples as you need to try out.

c) (voluntary) Can you prove your conjecture? (don't spend a lot of time if it seems difficult)

(SORTING COMPLEXITY)

Many common sorting algorithms are based on the idea of pairwise comparisons (insertsort, mergesort, quicksort etc.). The time complexity of such algorithms are often O(n ^ 2) or O(n log n) (better), which means that the number of steps such an algorithm may need in the worst case is proportional to n log n, where n is the number of items to be sorted. This is often written using the shorthand notation O(n log n) where O means "order of", sometimes called **big O notation**. We will here consider a slightly more abstract result, namely that any comparison-based algorithm will need at least n log n comparisons in the worst case. Read and understand the theorem and the proof. Explain the main ideas.

One version of the proof

Comparison sort in general

Please search as you like if you need more links about this - include any references you have used.

(investigating the world)

(BALANCING CHEMICAL REACTIONS)

We will here consider a common type of problem where a reaction is known in terms of which compounds are involved, but where the actual number of molecules is not. For example, consider the reaction for burning propane (e.g. in a propane cooker):

$C_3H_8 + O_2 \longrightarrow CO_2 + H_2O$

a) Model the problem as a system of diophantine equations, i.e. equations where the variables must be integers.

b) Solve with the Mathematica function `FindInstance`.

(Another similar problem is when you know the weight percentage of the atomic elements in a compound, and you want to determine the molecular formula for the compound)

(designing)

(PROJECT PLANNING PROBLEM)

Industrial projects where many people are involved are usually split into smaller tasks that are scheduled with their own starting time, duration and deadline. Several tasks can be ongoing in parallel, but if a task is dependent on other tasks it cannot begin until these tasks are completed. In this problem we will consider how a directed network can be used for planning and management of such projects.

a) Explain how a directed graph can be used to model a number of tasks in a project. Each task is assumed to have a

given duration and may be dependent on the completion of other tasks.

b) An important question is the minimum possible time to complete the project. For this purpose, determine how a shortest path algorithm can be used to find the critical path = the sequence of tasks that determines the minimum total length of the project. Hint: 1) make up a simple example and you will understand what we are asking for. 2) make sure you understand what problem it is that you want to solve, 3) only thereafter think about how to model your problem as a shortest path problem. Also make sure you did a) before you begin with b).

c) (voluntary) Suggest a linear programming model for finding the critical path. Hint: there is a more direct way to do this than first modelling as a shortest path and then as an LP.

d) In practice it may be the case that the exact duration of the tasks is not known. Suggest how the model could be extended to handle this. What useful things could be done with such an extended model and how? How easy would such a model be to use?

(RSA CRYPTOSYSTEM)

Explain in your own words **the RSA cryptosystem** and the main ideas behind. Try to do it to provide easy understanding of the main ideas, and also in sufficient detail so that someone who understands your explanation should be able to implement it. Feel free to search and use any sources - just list the references you have used. This is a **nice more complete explanation**. For modulo arithmetic in general, see the mathematical knowledge section below.

(GRAPH FOR PUBLIC TRANSPORT ROUTING)

(voluntary) Public transport operators often provide a web page or app to calculate the fastest way to travel from one point to another. This problem can be modelled as a shortest path problem for a directed weighted graph, for which efficient algorithms are available (shortest in this case refers to time). However, the question here is how to create/model the graph on which the shortest problem is to be solved. Try to give a general answer and also illustrate with a simple example.

(thinking)

(GOOD EXPLANATIONS)

a) Have a look at some **solutions to the problems in the course**, explained in a simple but reasonably good way. Note the flow of the explanations and compare with your

own explanations. You are here encouraged to not mainly focus on the answer itself but rather on how it is explained. What are your observations?

b) Can you relate the flow of an argument or explanation to some mathematical concept in this module?

c) How would you say that reasoning and explaining is related?

(mathematical knowledge)

(INTEGERS, SETS, SEQUENCES, GRAPHS, TREES)

These are the most basic discrete representations. I think you already have an understanding of integers, sets and sequences. For graphs you can read the link below, or look for other links about graphs. A tree in graph theory is an undirected graph with no cycles.

Graphs - Wikipedia

(MODULAR ARITHMETIC)

This is for example used in the RSA cryptosystem.

Modular arithmetic - Wikipedia

(finally...)

(SCHEDULING OF FINAL MEETING)

Every group will participate in a final meeting to discuss final reports and grading. Meetings will take place on Wednesday October 31, Thursday November 1 and Friday November 2 (2018). Clearly state when you are available, so that we can schedule your group appropriately. Please answer this question even if you do not have any constraints, so that we know this for sure. (Wednesday gives less time to read the reports, but if there are groups who have a particular reason to have the meeting then, it may be possible.)

(SELF-CHECK)

- Have you answered all questions to the best of your ability?
- Is the required information on the front page, file name correct etc.?
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If you pass the self-check, simply write "Self-check passed!". Otherwise, fix your submission before you submit - do not submit an incomplete module! You can receive personal help and/or a short extension if you contact a supervisor.

Remember to confirm your successful self-check!