5 Mostly dynamic systems

Depending on your background, some of these exercises may be easy, while others may be more difficult. We simply ask you to do your best with these problems, within the limits of the time allocated for the course.

Read **these reminders** before you begin!

(keeping track)

(investigating the abstract)

(ACHILLES AND THE TORTOISE)

- Have you heard about Achilles and the tortoise?
- No, what of it?
- Well, it is the hero Achilles and the tortoise. The tortoise says that he can win a race against Achilles if he just gets a head start.
- Why, that sounds crazy, the tortoise must be much slower.
- Yes, but listen. When the race starts Achilles is say 15 metres behind the tortoise. And then the race begins.
- Ok?
- So then they both begin to run, Achilles will quickly run these 15 meters to the place where the tortoise started, but then the tortoise has also moved ahead say 1 metre.
- Yes,…
- So the tortoise is still ahead. And when Achilles has run another metre to where the tortoise was before, the tortoise has moved yet some distance forward. And this repeats itself, since whenever Achilles runs to the point where the tortoise was before, the tortoise will have moved ahead yet again. So Achilles will never be able to overtake the tortoise!
- But this is nonsense. Of course Achilles will win - he runs faster!
- So it may seem, but what about the argument?
- I don’t care much about arguments, just use your common sense and you will be fine. Forget arguments, they are not to be trusted. It’s practice that counts, every sensible person knows that.
- But I find it interesting. I want to think about it. There must be something wrong with the argument, but I

haven't yet figured out what it is. I think it can be explained and that it will all agree with common sense, this is what I am using when I am thinking. And I am not going to give up until I really understand…

- Well, good luck, I'll go and watch a movie.

Can you sort this out? Is it possible to discover any interesting mathematics from this? Investigate and explain as well as you can!

(investigating the world)

(THROW BALL)

Do the following very simple theoretical experiment. Draw the trajectory of a thrown ball in the following way.

Start in (0,0) and draw a dot there.

Now work in steps:

- in the next step move 4 steps to the right, and 4 steps up. Draw a dot.
- then move 4 steps to the right and 3 steps up.
  (This is because the gravity of the earth changes the vertical speed, but not the horizontal speed. So decrease the vertical speed by one in every step!)
- continue with the same pattern and see what happens!

b) (voluntary) How could you make a more accurate simulation? (as always, a more accurate simulation may give a slightly different result)

(WHALES AND KRILL)

Here we will investigate a simple model for the population dynamics of whales and krill, where krill is assumed to be the main source of food for the whales. We consider the following model, described by the differential equations

$k' = (a-bw)\,k$

$w' = (-m+nk)\,w$

where k is the krill population (as a function of the time in years), w is the whale population, and a, b, m and n are positive constants. The two equations describe the rate of change k' for the krill, and the rate of change w' for the whales - also called the derivatives of k and w, with respect to the time. Together, the two equations determine the dynamic behaviour of this system, for any given initial values for k and w.

a) Investigate the program **krill.py** for simulating this system. Make sure you fully understand the program, and how the differential equations written above are used in the simulation. Study the behaviour of the system, and try to explain what you see and why. It is a good idea to enter different initial values to see what happens. There is also a second so called parametric plot that you can activate and look at.

b) Can you find any equilibrium points for this system with the help of the simulation and some thinking?
c) (voluntary) Can you establish any equilibrium points analytically?
d) Investigate the effect of krill fishing on these populations. To model this, we can add a term -rk to the equation for k', where r<a. Try out different values of r, simulate and discuss your observations.
e) (voluntary) Have a look at how you can do essentially the same thing in Mathematica with the built-in command `NDSolve`

```
sol = NDSolve[{k'[t] == (0.2 - 0.0001
w[t]) k[t], w'[t] == (-0.5 + 0.000001
k[t]) w[t], k[0] == 700000,  w[0] ==
3000}, {k, w}, {t, 100}];
Plot[Evaluate[k[t] /. sol], {t, 0, 100}]
Plot[Evaluate[w[t] /. sol], {t, 0, 100}]
```

(designing)

(BOUNCING BALLS PROGRAM)

This Java program simulates bouncing balls in a box. First run the program. Then read the program and try to understand it. Explain the mathematics involved. The entire model is in the method `step` in `Model.java` so focus your efforts there.

(TEMPERATURE CONTROL SYSTEM)

The program **control.py** simulates a simple model for the temperature in a room. Initially the model implements a 1000W heater fan with simple thermostat regulation. Due to particular storage requirements for a special kind of paint, it is desired to have an excellent temperature regulation at 17 degrees Celsius. If possible, this should work in the range of -5 to +25 degrees of outside temperature.

a) Try to understand the program, run it and make any observations.
b) Improve the design of the regulator to meet the objectives as well as possible. The possibilities you have in your design is to dimension the max power of the

heating fan, and to implement the regulator, which is allowed to set the power of the fan to any value once every minute.

(thinking)

(MATHEMATICAL MODELLING OF DYNAMIC SYSTEMS)

Now when you you see the similarity between the program in the previous module for generating some well-known mathematical functions using the rate-of-change/derivative, and these different simulations of real-world systems, what are your thoughts?

(SOCRATIC QUESTIONING)

In this course an important method of supervision is to ask questions, and often you are then able to answer these questions yourselves. Now temporarily place yourself in the role of the supervisor, and suggest one or a few good supervision questions in each of the following student situations:

a) We have looked at this table for a while, but we haven't learned any method to find a curve for such tables (the problem with the planets, can be solved without knowing about the least square method).

b) We tried with the log function, and think it is about right but maybe not perfect. Is that the answer? (planets again)

d) We don't know how to solve this problem, and are stuck. What should we do? (any problem)

c) We have written a constraint here - is it correct? (emergency care problem)

e) We just want to check if we are on the right track… (any problem)

f) Why do you think I am asking you to suggest these questions? (this is not an imagined student question but a question directly to you!)

(mathematical knowledge)

(DERIVATIVES)

See the lecture notes for the introduction to this module. I added a slide at the end that was not in the lecture, to clarify how the simulation is a process of *integration*, i.e. to go from the derivative to the function itself.

There are of course many books and websites that describe what a derivative is. Most begin with a formal definition of the derivative, and then go on to analytically calculate the derivatives of different functions. An similarly with integrals. Nothing wrong with that, but in this course we have chosen to focus mostly on the concept of the derivative, and then directly engage in how this fundamental can be used in modelling and simulating different dynamic systems. Of course, for algebraic calculations and other purposes, some of this ordinary math can be needed.

(finally…)

(SELF-CHECK)

- Have you answered all questions to the best of your ability?
- Is the required information on the front page, file name correct etc.?
- Anything else you can easily check?

If you pass the self-check, simply write "Self-check passed!”. Otherwise, fix your submission before you submit - do not submit an incomplete module! You can receive personal help and/or a short extension if you contact a supervisor.

Remember to confirm your successful self-check!