Some comments on mathematical thinking (mid-course)

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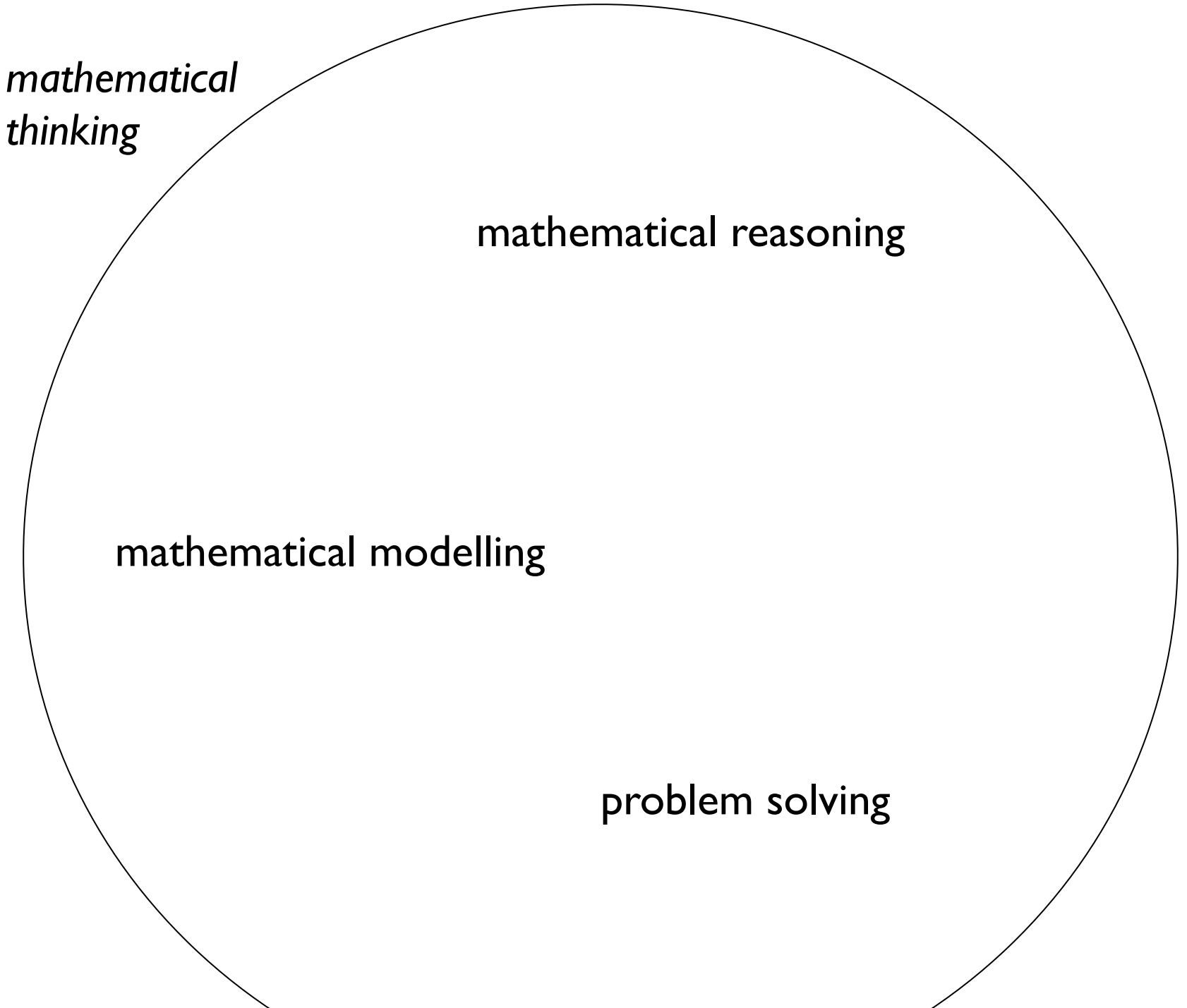

mathematical thinking
mathematical reasoning
mathematical modelling
problem solving

mathematical reasoning

Some experiences

be clear about when you understand and when you don't understand

be clear about what you know, what you believe and what you don't know.

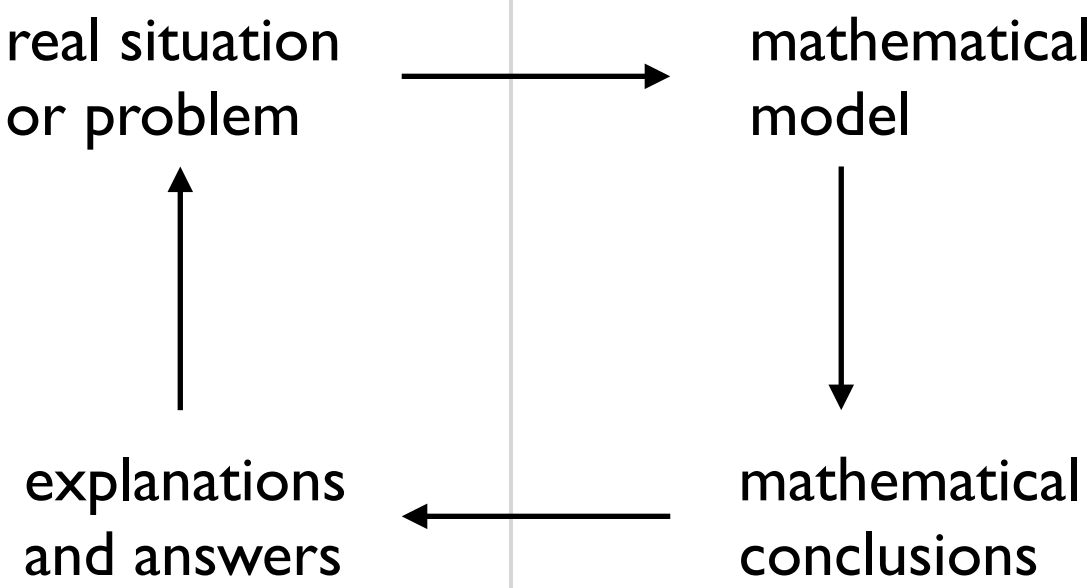
distinguish between plausible and deductive reasoning

distinguish between knowing and understanding why

be extremely careful not to say anything that is factually or logically wrong (but you can still say a lot if you do it in the right way!)

mathematical modelling

The modelling cycle



Some experiences

real-world problems are often not particularly well-defined, and you need to create your own more precise interpretation

"ill-structured problems"

distinguish between between the real problem and the mathematical model and keep both in mind

qualitative and quantitative understanding!

Think about the real situation and your real-world knowledge and integrate whenever appropriate!

What is good enough?

A critical decision: selecting the modelling approach!

function/equation?

optimization problem?

geometric model?

dynamic model?

probabilistic model?

discrete model?

we mostly consider different kinds of models different weeks!

problem solving

What is problem solving?

"You are engaged in problem solving when you are trying to achieve something, and you do not know a straightforward way to do so."

(A. Schoenfeld)

"what to do when you don't know what to do"

Why problem solving?

It's the *variation*!

(Problems come in infinite variations - it will never be sufficient to learn a finite set of given methods)

Work in small manageable steps…

don't expect to think all the way to the solution!

If you want to move this bookshelf to another wall you need to work in small steps!

Typical workflow - intermediate problems

1. Understand the problem

2. Make a plan

3. Carry out the plan

4. Look back (check your result,
reflect on the process, ...)

(Polya)

Typical workflow – more difficult problems

1. Understand the problem
2. Make a plan
3. Carry out the plan
4. Look back

investigate for deeper understanding, define clearly

explore different approaches, begin with something simple!

when you fail you learn and go back

Continuously reflect, go back and revise, manage your time

You will have to struggle!

This requires a lot of self-awareness!

An important problem solving technique:
asking questions!

a question creates a subproblem!

Heuristics = generally useful questions!

Heuristic: a specific approach "to understand a problem better or to make progress towards its solution" (Schoenfeld)

Can you …?

simplify, specialize, create an example, split in parts, draw a figure, find a related problem, consider extreme cases, guess and check, systematically test, change representation, … (lots!)

I find it useful to think of heuristics as questions rather than as techniques since they are not algorithms!

What is needed for successful problem solving? (Schoenfeld)

Resources (knowledge of different kinds)

Heuristics ("tricks of the trade")

Self-regulation (monitoring and control, "self-awareness")

Appropriate attitudes and expectations (your "belief system")

(in-class comments 2018)

examples good for understanding and explaining

discuss!

conjecture first!

assumptions allow you to continue

work in small steps

not just right or wrong

connect reality to model

understand the problem

many solutions to a problem

try simple solution first!

ask questions!

you only need to be right at the end

be careful!

time management

Some experiences
(suggested by Dag 2017)

try things out - think and
learn along the way!

try something
simple first!

explore
alternatives

understand the
problem

investigate the
problem for deeper
understanding

many ways to
many solutions

failure is normal
and a learning
opportunity

END